Answer on Question #83852 – Math – Differential Equations

Question

A tank contains 100 liters of pure water. Brine that contains 0.1 kg of salt per liter enters the tank at a rate of 10 L/min. The solution is kept thoroughly mixed and drains from the tank at the same rate. How much salt is the tank after 6 minutes?

Solution

Let $x(t)$ be the mass of salt, in kilograms, that is in the tank at time t. Since the tank is initially filled with fresh water we know that $x(0) = 0$.
An expression for $x'(t)$ will be given as the rate salt enters the tank minus the rate salt leaves the tank (in kilograms per minute).

$$x'(t) = \frac{dx}{dt} = (rate\ in) - (rate\ out) =$$

$$= \left(\frac{0.1\ kg}{1\ L}\right)\left(10\frac{L}{min}\right) - \left(\frac{x(t)kg}{100\ L}\right)\left(10\frac{L}{min}\right)$$

$$\frac{dx}{dt} = \frac{10 - x}{10}$$

$$\frac{dx}{10 - x} = \frac{1}{10}dt$$

$$\int \frac{dx}{10 - x} = \frac{1}{10}\int dt$$

$$-\ln|10 - x| = \frac{1}{10}t + C$$

$$x(0) = 0: C = -\ln 10$$

$$\ln|10 - x| = \ln 10 - \frac{1}{10}t$$

$$|10 - x| = 10e^{-t/10}$$

Since $x < 10$

$$10 - x = 10e^{-t/10}$$

$$x = 10 - 10e^{-t/10} = 10\left(1 - e^{-t/10}\right)$$

Then

$$x(6) = 10\left(1 - e^{-6/10}\right) = 10(1 - e^{-0.6})\ (kg)$$

$$10(1 - e^{-0.6})\ kg \approx 4.512\ kg$$

Answer: $10(1 - e^{-0.6})\ kg \approx 4.512\ kg$