Answer on Question #83832 – Math – Real Analysis

Question

Show that n!<n^n, n>1, by induction.

Solution

We shall prove it by the method of mathematical induction.

1. Take n = 2 and substitute the inequality, we get: $2!=1\times2=2<2^2=2\times2=4$, i.e. 2<4, therefore the inequality holds.
2. Let it be true for n = k, where $k\geq2$, i.e. $k!<k^k$.
3. Let us prove the fulfillment of the inequality for n = k + 1, where $k\geq2$.

We need to show $(k+1)!<(k+1)^{k+1}$, or $k!(k+1)<(k+1)^{k+1}$, or $k!<(k+1)^k$. Transform the right hand side of the inequality as follows: $(k(1+\frac{1}{k}))^k=k^k(1+\frac{1}{k})^k$. We know that $\lim_{k\to\infty}\left(1+\frac{1}{k}\right)^k$=e, i.e. always $1<(1+\frac{1}{k})^k<e$. And, at the same time, we have assumed that $k!<k^k$, then k! also will be less than $k^k(1+\frac{1}{k})^k$, and then $(k+1)!<(k+1)^{k+1}$.

4. By the method of mathematical induction, the inequality $n!<n^n$ holds for all n>1.