Answer on Question #83825 – Math – Statistics and Probability

Question

A cab taxi company has 12 Santro and 8 Alto cars. If these taxi cabs are in the workshop for repairs and a Santro is as likely to be in for repair as an Alto. What is the probability that,

i) 3 of them are Santro and 2 are Alto.

ii) At least 3 of them are Senators

iii) All the 5 are of the same make?

Solution

The order of the cars chosen in the workshop for repairs is not important, so this situation is a combination of $(12+8)$ cars taken 5 at a time, and is thus equal to

$$N = C(20,5) = \frac{20!}{5!\,(20-5)!} = \frac{20(19)(18)(17)(16)}{1(2)(3)(4)(5)} = 15504$$

i) 3 of them are Santro and 2 are Alto.

Three Santro cars can be taken in the workshop for repairs in $C(12,3)$ ways, and two Alto cars can be taken in the workshop for repairs in $C(8,2)$ ways

$$C(12,3) \times C(8,2) = \frac{12!}{3!\,(12-3)!} \times \frac{8!}{2!\,(8-2)!} = \frac{12(11)(10)}{1(2)(3)} \times \frac{8(7)}{1(2)} = 6160$$

The probability that 3 of them are Santro and 2 are Alto is

$$P(3\&2) = \frac{C(12,3) \times C(8,2)}{C(20,5)}$$

$$P(3\&2) = \frac{\frac{12!}{3!\,(12-3)!} \times \frac{8!}{2!\,(8-2)!}}{\frac{20!}{5!\,(20-5)!}} = \frac{6160}{15504} = \frac{385}{969} \approx 0.3973$$

ii) At least 3 of them are Senators

$$P(3\&2) + P(4\&1) + P(5\&0) =$$

$$= \frac{C(12,3) \times C(8,2)}{C(20,5)} + \frac{C(12,4) \times C(8,1)}{C(20,5)} + \frac{C(12,5) \times C(8,0)}{C(20,5)}$$

$$C(12,3) \times C(8,2) = \frac{12!}{3!\,(12-3)!} \times \frac{8!}{2!\,(8-2)!} = \frac{12(11)(10)}{1(2)(3)} \times \frac{8(7)}{1(2)} = 6160$$

$$C(12,4) \times C(8,1) = \frac{12!}{4!\,(12-4)!} \times 8 = \frac{12(11)(10)(9)}{1(2)(3)(4)} \times 8 = 3960$$

$$C(12,5) \times C(8,0) = \frac{12!}{5!\,(12-5)!} \times 1 = \frac{12(11)(10)(9)(8)}{1(2)(3)(4)(5)} = 792$$

$$P(3\&2)+P(4\&1)+P(5\&0)=\frac{6160+3960+792}{15504}=\frac{10912}{15504}=\frac{682}{969}\approx 0.7038$$

iii) All the 5 are of the same make

$$P(5\&0)=\frac{C(12,5)\times C(8,0)}{C(20,5)}=\frac{\frac{12!}{5!\,(12-5)!}\times 1}{15504}=\frac{792}{15504}$$

$$P(0\&5)=\frac{C(12,0)\times C(8,5)}{C(20,5)}=\frac{1\times\frac{8!}{5!\,(8-5)!}}{15504}=\frac{56}{15504}$$

$$P(5\&0)+P(0\&5)=\frac{792}{15504}+\frac{56}{15504}=\frac{848}{15504}=\frac{106}{1938}\approx 0.0547.$$