Let's prove product formula using logarithms. Let $f = uv$ and suppose u and v are positive functions of x. Then

$$\ln f = \ln(u \cdot v) = \ln u + \ln v.$$

Differentiating both sides:

$$\frac{1}{f}\frac{df}{dx} = \frac{1}{u}\frac{du}{dx} + \frac{1}{v}\frac{dv}{dx}$$

and so, multiplying the left side by f, and the right side by uv,

$$\frac{df}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}.$$

2). Suppose $f(x) = g(x)/h(x)$, where $h(x) \neq 0$ and g and h are differentiable.

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\frac{g(x+\Delta x)}{h(x+\Delta x)} - \frac{g(x)}{h(x)}}{\Delta x}$$

We pull out the $1/\Delta x$ and combine the fractions in the numerator:

$$= \lim_{\Delta x \to 0} \frac{1}{\Delta x}\left(\frac{g(x+\Delta x)h(x) - g(x)h(x+\Delta x)}{h(x)h(x+\Delta x)}\right)$$

Adding and subtracting $g(x)h(x)$ in the numerator:

$$= \lim_{\Delta x \to 0} \frac{1}{\Delta x}\left(\frac{g(x+\Delta x)h(x) - g(x)h(x) - g(x)h(x+\Delta x) + g(x)h(x)}{h(x)h(x+\Delta x)}\right)$$

We factor this and multiply the $1/\Delta x$ through the numerator:

$$= \lim_{\Delta x \to 0} \frac{\frac{g(x+\Delta x)-g(x)}{\Delta x}h(x) - g(x)\frac{h(x+\Delta x)-h(x)}{\Delta x}}{h(x)h(x+\Delta x)}$$

Now we move the limit through:

$$= \frac{\lim_{\Delta x \to 0}\left(\frac{g(x+\Delta x)-g(x)}{\Delta x}\right)h(x) - g(x)\lim_{\Delta x \to 0}\left(\frac{h(x+\Delta x)-h(x)}{\Delta x}\right)}{h(x)\lim_{\Delta x \to 0} h(x+\Delta x)}$$

By the definition of the derivative, the limits of difference quotients in the numerator are derivatives. The limit in the denominator is $h(x)$ because differentiable functions are continuous. Thus we get:

$$= \frac{g'(x)h(x) - g(x)h'(x)}{[h(x)]^2}.$$