

Answer on Question #83724 – Math – Statistics and Probability

Question

According to a survey by the Administrative Management Society, one-half of U.S. companies give employees 4 weeks of vacation after they have been with the company for 15 years. Find the probability that among 6 companies surveyed at random, the number that give employees 4 weeks of vacation after 15 years of employment is (a) anywhere from 2 to 5; (b) fewer than 3.

Solution

Let X be the random variable which denotes the number of companies that give employees 4 weeks of vacation after 15 years of employment, among 6 companies surveyed at random. Then $X \sim B(n, p)$. Given that $n = 6, p = 0.5$.

By Binomial Probability law

$$P(X = x) = C_x^n p^x (1 - p)^{n-x} = \binom{n}{x} p^x (1 - p)^{n-x}$$

(a) anywhere from 2 to 5

$$\begin{aligned} P(2 \leq X \leq 5) &= P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5) = \\ &= \binom{6}{2} (0.5)^2 (1 - 0.5)^{6-2} + \binom{6}{3} (0.5)^3 (1 - 0.5)^{6-3} + \binom{6}{4} (0.5)^4 (1 - 0.5)^{6-4} + \\ &+ \binom{6}{5} (0.5)^5 (1 - 0.5)^{6-5} = \frac{1}{64} (15 + 20 + 15 + 6) = \frac{7}{8} = 0.875 \end{aligned}$$

(b) fewer than 3

$$\begin{aligned} P(X < 3) &= P(X = 0) + P(X = 1) + P(X = 2) = \\ &= \binom{6}{0} (0.5)^0 (1 - 0.5)^{6-0} + \binom{6}{1} (0.5)^1 (1 - 0.5)^{6-1} + \binom{6}{2} (0.5)^2 (1 - 0.5)^{6-2} = \\ &= \frac{1}{64} (1 + 6 + 15) = \frac{11}{32} = 0.34375 \end{aligned}$$

Answer: a) 0.875; b) 0.34375.