Answer on Question #83693 – Math – Analytic Geometry

Question

Find the equation of the plane through the line $\frac{x-2}{2}=\frac{y-3}{3}=\frac{z-4}{5}$ and parallel to x-axis?

Solution

From the equation of line

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}, (1)$$

we have

$x_2-2=2;$ $y_2-3=3;$ $z_2-4=5;$

$x_2=4.$ $y_2=6.$ $z_2=9.$

Because the line lies on the plane, the plane also contains points (2, 3, 4) and (4, 6, 9).

Consider the plane equation

$$Ax+By+Cz+D=0 \ (2)$$

or

$$\frac{A}{D}x+\frac{B}{D}y+\frac{C}{D}z+1=0 \text{ if } D\neq 0 \ (3)$$

($A=0$ because the plane parallel to x-axis) make such a system:

$$\begin{cases} 3\frac{B}{D}+4\frac{C}{D}+1=0\ ; \\ 6\frac{B}{D}+9\frac{C}{D}+1=0. \end{cases}$$

$$A=0,\ \frac{B}{D}=-\frac{5}{3} \text{ and } \frac{C}{D}=1 \ (4)$$

From (3), (4) it follows that

$$-\frac{5}{3}y+z+1=0 \text{ if } D\neq 0, \text{ hence } 5y-3z-3=0.$$

Answer: $5y-3z-3=0$ is the equation of the plane.