Question

Find the equation of the plane through (2,3,-4) and (1,-1,3) and parallel to x-axis?

Solution

Assume that Ax + By + Cz + D = 0 is an equation of the plane.

The plane is parallel to x- axis, hence

$$A = 0$$
 (1)

The points (2,3,-4), (1,-1,3) lie on the plane, hence

$$\begin{cases} 3B - 4C + D = 0, \\ -B + 3C + D = 0. \end{cases}$$

$$\begin{cases} D = 4C - 3B, \\ D = B - 3C. \end{cases}$$

$$4C - 3B = B - 3C, \\ -4B = -7C, \\ C = \frac{4}{7}B, \text{ (2)} \end{cases}$$

$$D = 4C - 3B = 4 * \frac{4}{7}B - 3B = \frac{16}{7}B - 3B = \frac{16-21}{7}B = -\frac{5}{7}B, \\ D = -\frac{5}{7}B, \text{ (3)} \\ By + cz + d = 0, \\ By + \frac{4}{7}Bz - \frac{5}{7}B = 0, \text{ (4)} \end{cases}$$

lf

B = 0, (5)

then it follows from (2), (3), (5) that

$$C = 0, D = 0$$
 (6)

In case of (5) taking (1), (5), (6) into account one gets that all coefficients in the equation

Ax + By + Cz + D = 0

are zero, which is impossible, therefore

$$B \neq 0$$
 (7)

Using (7) divide the equation (4) through by $B \neq 0$.

Then

$$7y + 4z - 5 = 0$$

is the equation of the plane through (2,3,-4), (1,-1,3) and parallel to x- axis.

Answer: 7y + 4z - 5 = 0.