Answer on Question #83676 – Math – Analytic Geometry

Question

Find the angle between the line joining the points (3, -4, -2) and (12, 2, 0) and the plane 3x-y+z=1

Solution

Equation of a line that passes through two points A(3,-4,-2) and B(12, 2, 0):

AB: $\frac{x-x_A}{x_B-x_A} = \frac{y-y_A}{y_B-y_A} = \frac{z-z_A}{z_B-z_A}$;

AB: $\frac{x-3}{12-3} = \frac{y+4}{2+4} = \frac{z+2}{0+2}$;

AB: $\frac{x-3}{9} = \frac{y+4}{6} = \frac{z+2}{2}$.

Then $\overrightarrow{AB}(9,6,2)$ is a direction vector of the straight line.

If the plane is given by the equation 3x-y+z=1, then a normal vector of the plane is $\vec{n}(3,-1,1)$.

The angle between the straight line and the normal vector is calculated by the formula:

$$\cos(\overrightarrow{\mathrm{AB}};\vec{\mathrm{n}}) = \frac{\overrightarrow{AB}\cdot\vec{n}}{|\overrightarrow{AB}|\cdot|\vec{n}|};$$

$$\cos(\overrightarrow{\mathrm{AB}};\vec{\mathrm{n}}) = \frac{9\cdot3+6\cdot(-1)+2\cdot1}{\sqrt{9^2+6^2+2^2}\cdot\sqrt{3^2+(-1)^2+1^2}};$$

$$\cos(\overrightarrow{\mathrm{AB}};\vec{\mathrm{n}}) = \frac{27-6+2}{\sqrt{81+36+4}\cdot\sqrt{9+1+1}};$$

$$\cos(\overrightarrow{\mathrm{AB}};\vec{\mathrm{n}}) = \frac{23}{\sqrt{121}\cdot\sqrt{11}};$$

$$\cos(\overrightarrow{\mathrm{AB}};\vec{\mathrm{n}}) = \frac{23}{11\sqrt{11}};$$

$$\cos(\overrightarrow{\mathrm{AB}};\vec{\mathrm{n}}) \approx 0.63;$$

The angle between the straight line and the plane is calculated by the formula:

$$sin\varphi = |cos(\overrightarrow{\mathrm{AB}};\vec{\mathrm{n}})|;$$
$$sin\varphi = 0.63;$$
$$\varphi \approx 50^{\circ}54'.$$

Answer: $\varphi \approx 50^{\circ}54'$.

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