Answer on Question #83608 – Math – Calculus

Question

Let $F(x) = \begin{cases} -1, & x < 0 \\ 0, & x = 0 \\ 1, & x > 0 \end{cases}$. Find $\lim\limits_{x \to 0} F(x)$.

Solution

Let $x_n = -\frac{1}{n}, n \in \mathbb{N}$, is a sequence of real numbers. Then $\lim\limits_{n\to\infty} x_n = 0$, and

$L_1 = \lim\limits_{x_n \to 0} f(x_n) = \lim\limits_{n\to\infty} f(x_n) = \lim\limits_{n\to\infty} (-1) = -1.$

Let $y_n = \frac{1}{n}, n \in \mathbb{N}$, is another sequence of real numbers. Then $\lim\limits_{n\to\infty} y_n = 0$, and

$L_2 = \lim\limits_{y_n \to 0} f(y_n) = \lim\limits_{n\to\infty} f(y_n) = \lim\limits_{n\to\infty} (1) = 1.$

Since the limits L_1 and L_2 are not equal, the limit of the function $F(x)$ as x tends to 0 does not exist. It is undefined by Heine's definition of the limit of a function.

Answer: $\lim\limits_{x\to 0} F(x)$ does not exist.