Answer on Question #83542 – Math – Analytic Geometry
Question

How are the points (3, 5) and (5, 3) different?

Solution

Two points are said to be symmetrical with respect to a given point when the given point bisects the line joining the two points. The given point is called the center of symmetry.

Two points are said to be symmetrical with respect to a given line when the given line is the perpendicular bisector of the line joining the two points. The given line is called the axis of symmetry.

We have two points $A(3,5)$ and $B(5,3)$. Find the coordinates of the point C bisecting a line segment AB from $(3,5)$ to $B(5,3)$

$$x_C = \frac{x_A + x_B}{2}, y_C = \frac{y_A + y_B}{2}$$

$$x_C = \frac{3+5}{2} = 4, y_C = \frac{5+3}{2} = 4, point\ C(4,4)$$

Two points $A(3,5)$ and $B(5,3)$ are symmetrical with respect to point $C(4,4)$.

Find the slope of the line passing through the points $A(3,5)$ and $B(5,3)$

$$slope_1 = m_1 = \frac{y_B - y_A}{x_B - x_A} = \frac{3-5}{5-3} = -1$$

If two lines with slopes m_1 and m_2 are perpendicular, then

$$m_1 m_2 = -1$$

Find the slope of the line which is perpendicular to the line AB

$$slope_2 = m_2 = -\frac{1}{m_1} = -\frac{1}{-1} = 1$$

The equation of the line perpendicular to the line AB

$$y = x + b$$

Find the equation of the line perpendicular to the line AB and passing through the point $C(4,4)$

$$4 = 4 + b => b = 0$$

$$y = x$$

Therefore, two points $A(3,5)$ and $B(5,3)$ are symmetrical with respect to the line $y = x$.

Find the distance d_{AB}

$$d_{AB} = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2} = \sqrt{(5-3)^2 + (3-5)^2} = 2\sqrt{2}$$