

Multiplication of matrices is not commutative (in general):

$$AB \neq BA$$

First of all because AB and BA may not be simultaneously define.

For multiplication AB number of columns in A must be equal to the number of rows in B. So we will have problem if we want multiply BA.

Example 1

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \quad B = \begin{pmatrix} 5 & 6 & 7 \\ 8 & 9 & 10 \end{pmatrix}$$

AB: in A - 2 columns, in B - 2 rows. The result AB of their multiplication is a 3×3 matrix

BA: in B – 3 columns, in A – 2 rows. So multiplication BA not defined.

If AB and BA both simultaneously defined, we have next situation:

Example 2:

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \quad B = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

Then

$$AB = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a + 2c & b + 2d \\ 3a + 4c & 3b + 4d \end{pmatrix}$$

But

$$BA = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} a + 3b & 2a + 4b \\ c + 3d & 2c + 4d \end{pmatrix}$$

$$AB \neq BA$$

So multiplication of matrices is not commutative.