

## Answer on Question #83464 – Math – Statistics and Probability

### Question

Packets of milk powder produced by a machine were found to have a normal distribution with a mean mass of 650g and a standard deviation of 10g.

**(a)** Find the probability that a packet selected at random will have a mass between 620g and 655g.

**(b)** If 500 packets are selected at random, how many of them will have a mass of more than 660g?

**(c)** It is found that 10% packets of milk powder will have a mass of less than  $k$  grams. Calculate  $k$ .

### Solution

**(a)**

If  $X$  is a normally distributed random variable with mean  $\mu$  and standard deviation  $\sigma$ , then the probability that a randomly chosen value of  $x$  will be greater than  $a$ , and less than  $b$ , is equal to

$$P(a, b | \mu, \sigma) = \Phi\left(\frac{b-\mu}{\sigma}\right) - \Phi\left(\frac{a-\mu}{\sigma}\right),$$

where  $\Phi(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^z e^{-\frac{t^2}{2}} dt$  is the cumulative distribution function of the standard normal distribution.

Thus,

$$\begin{aligned} P(620, 655 | \mu=650, \sigma=10) &= \Phi\left(\frac{655-650}{10}\right) - \Phi\left(\frac{620-650}{10}\right) = 0.691462 - 0.00135 = \\ &= 0.690113 \end{aligned}$$

$$\begin{aligned} (\text{In Excel } \Phi\left(\frac{655-650}{10}\right) - \Phi\left(\frac{620-650}{10}\right) &= \text{NORMDIST}(655;650;10;1) - \\ \text{NORMDIST}(620;650;10;1)) \end{aligned}$$

**(b)**

This value is calculated by the formula

$$N = n * P(x > c | \mu, \sigma) = n(1 - \Phi\left(\frac{c - \mu}{\sigma}\right)),$$

where  $n = 500$ ,  $c = 660$ .

Thus,

$$N = 500 * (1 - \Phi\left(\frac{660 - 650}{10}\right)) = 500 * (1 - 0.841345) = 500 * 0.158655 = 79$$

$$(\text{In Excel } \Phi\left(\frac{660 - 650}{10}\right) = \text{NORMDIST}(660; 650; 10; 1))$$

**(c)**

The value of  $k$  is found from the formula  $\Phi\left(\frac{k - \mu}{\sigma}\right) = 0.1$  :

$$\frac{k - \mu}{\sigma} = \Phi^{-1}(0.1);$$

$$k = \sigma \Phi^{-1}(0.1) + \mu;$$

$$k = 10 * (-1.28155) + 650 = 637$$

$$(\text{In Excel } \Phi^{-1}(0.1) = \text{NORMSINV}(0.1)).$$

**Answer:**

**(a)** The probability that a packet selected at random will have a mass between 620g and 655g is equal to 0.690113.

**(b)** If 500 packets are selected at random, that 79 of them will have a mass of more than 660g.

**(c)**  $k = 637\text{g}$