Answer on Question #83463 – Math – Statistics and Probability

Question

A population of 10,000 students has mean age 20 years old and standard deviation 4 years. Find the mean and standard error of sampling distribution of the sample mean $\bar{x}$ for sample sizes of 100.

Solution

The mean of the sampling distribution of means, denoted by $\mu_{\bar{x}}$ is given by $\mu_{\bar{x}} = \mu$, where μ is the mean of the population. And the standard error of the sampling distribution $\sigma_{\bar{x}}$ is determined by the standard deviation of the population σ, the population size N, and the sample size n in the equation below:

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \cdot \sqrt{\frac{N-n}{N-1}}$$

In the standard error formula, the factor $\sqrt{\frac{N-n}{N-1}}$ is called the finite population correction. When the population size is very large relative to the sample size, the finite population correction is approximately equal to 1. So the standard error formula can be approximated by:

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

As a general rule, it is safe to use the approximate formula $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$, when $n/N \leq 0.05$

The above formulas can be used for the normally distribution. According to the Central Limit Theorem for samples of size 30 or more, the sample mean is approximately normally distributed. In our case $n = 100 > 30$ so we can use the above formulas.

We have that the mean of the population $\mu = 20$, so $\mu_{\bar{x}} = 20$.

In our case $n = 100$, $N = 10{,}000$, $\sigma = 4$, $\frac{n}{N} = \frac{100}{10{,}000} = 0.001 < 0.05$, so we use the formula $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$.

$$\sigma_{\bar{x}} = \frac{4}{\sqrt{100}} = 0.4$$

Answer: $\mu_{\bar{x}} = 20$, $\sigma_{\bar{x}} = 0.4$.