

Answer on Question #83377 – Math – Discrete Mathematics.

Question

In a lottery, players win a large prize when they pick four digits that match, in the correct order, four digits selected by a random mechanical process. A smaller prize is won if only three digits are matched. What is the probability that a player wins the large prize? What is the probability that a player wins the small prize?

Solution

The probability that a player wins the large prize is 0.0001.

(In total, there are 10,000($10 \cdot 10 \cdot 10 \cdot 10$) different variants of the four digits chosen by the random mechanical process. With only one combination of four digits players win a large prize. Thus, the probability that a player wins the large prize is $1/10000=0.0001$).

The probability that a player wins the small prize is 0.0036.

(In total, there are 10,000($10 \cdot 10 \cdot 10 \cdot 10$) different variants of the four digits chosen by the random mechanical process. Players win a small prize, when only three digits are matched. The number of possible options when exactly three digits match: $\binom{4}{3} \cdot (10 - 1) = \frac{4!}{3! \cdot 1!} \cdot 9 = 4 \cdot 9 = 36$. (Here $\binom{4}{3}$ is the number of ways to choose 3 digits that will match; $(10-1)$ is the number of ways to select the remaining digit that will not match). With only 36 combination of four digits players win a small prize. Thus, the probability that a player wins the small prize is $36/10000=0.0036$).