Answer on Question #83337 – Math – Analytic Geometry

Question

(1) The scalar product of vectors $\boldsymbol{a}$ and $\boldsymbol{b}$, where θ is the angle between them, is

Solution

$a \cdot b = \|a\|\|b\| \cos\theta$

Question

(2) Determine the gradient of a straight line passing through the point $(1, 6)$ and $(-3, -3)$.

Solution

Gradient (slope) of a straight line

$$grad = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - 6}{-3 - 1} = \frac{9}{4}$$

Question

(3) Given a circle with centre at the origin, which passes through the point $(2, -1)$. Find its equation.

Solution

The equation of the circle with centre at the origin

$$x^2 + y^2 = R^2$$

Substitute

$$(2)^2 + (-1)^2 = R^2$$

$$R^2 = 5$$

The equation of the circle with centre at the origin, which passes through the point $(2, -1)$ is

$$x^2 + y^2 = 5$$

Question

(4) Find the unit vector in the direction of vector $\boldsymbol{b} = 3\boldsymbol{i} + 4\boldsymbol{j} - 5\boldsymbol{k}$

Solution

$$\|\boldsymbol{b}\| = \sqrt{(3)^2 + (4)^2 + (-5)^2} = 5\sqrt{2}$$

The unit vector in the direction of vector $\boldsymbol{b} = 3\boldsymbol{i} + 4\boldsymbol{j} - 5\boldsymbol{k}$

$$\boldsymbol{u} = \frac{\boldsymbol{b}}{||\boldsymbol{b}||}$$

$$\boldsymbol{u} = \frac{3\sqrt{2}}{10}\boldsymbol{i} + \frac{2\sqrt{2}}{5}\boldsymbol{j} - \frac{\sqrt{2}}{2}\boldsymbol{k}$$

Question

(5) A line AB passes through the point $P\ (3, -2)$ with gradient -2. Determine the equation of the line CD through P perpendicular to AB.

Solution

If two lines are perpendicular, then $grad_1 grad_2 = -1$.

$$grad_{CD} = -\frac{1}{grad_{AB}} = -\frac{1}{-2} = \frac{1}{2}$$

The equation of the line CD

$$y = \frac{1}{2}x + b$$

Substitute and find b

$$-2 = \frac{1}{2}(3) + b \Rightarrow b = -\frac{7}{2}$$

The equation of the line CD

$$y = \frac{1}{2}x - \frac{7}{2}$$

Question

(6) Determine the equation of a straight line passing through the point $(1, 0)$ and $(2, -3)$.

Solution

Gradient (slope) of a straight line

$$grad = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - 0}{2 - 1} = -3$$

The equation of the line

$$y = -3x + b$$

Substitute and find b

$$0 = -3(1) + b \Rightarrow b = 3$$

The equation of a straight line passing through the point $(1, 0)$ and $(2, -3)$.

$$y = -3x + 3$$

Question

(7) Determine the scalar product of vectors $2\boldsymbol{i}+3\boldsymbol{j}-5\boldsymbol{k}$ and $4\boldsymbol{i}+\boldsymbol{j}-6\boldsymbol{k}$

Solution

$$(2\boldsymbol{i}+3\boldsymbol{j}-5\boldsymbol{k})\cdot(4\boldsymbol{i}+\boldsymbol{j}-6\boldsymbol{k}) = 2(4)+3(1)+(-5)(-6) = 41$$

Question

(8) Find the equation of the line which is parallel to the line $2y + \ 3x = 3$ and passes through the midpoint of $(-2,3)$ and $(4,5)$.

Solution

If two lines are parallel, then they have the same gradient (slope).
Find the gradient slope

$$2y + \ 3x = 3 => y = -\frac{3}{2}x+\frac{3}{2}$$

$$grad = -\frac{3}{2}$$

The midpoint of two points

$$x_m = \frac{x_1+x_2}{2}\ , y_m = \frac{y_1+y_2}{2}$$

Substitute

$$x_m = \frac{-2+3}{2} = \frac{1}{2}\ , y_m = \frac{3+5}{2} = 4$$

Find the equation of the line

$$y = -\frac{3}{2}x + b$$

Substitute and find b

$$4 = -3\left(\frac{1}{2}\right) + b => b = \frac{15}{2}$$

The equation of the line which is parallel to the line $2y + \ 3x = 3$ and passes through the midpoint of $(-2,3)$ and $(4,5)$

$$y = -\frac{3}{2}+\frac{15}{2}$$

Question

(9) Find the centre and radius of each of the circle $x^2+y^2-2x-6y=15$

Solution

$$x^2+y^2-2x-6y=15$$

$$x^2-2x+1+y^2-6y+9-1-9=15$$

$$(x-1)^2+(y-3)^2=25$$

The equation of the circle with center at (h,k) and radius r

$$(x-h)^2+(y-k)^2=r^2$$

Therefore, we have the equation of the circle with the center $(1, 3)$ and radius 5.

Question

(10) Determine the direction cosines $[l, m, n]$ of the vector $3\boldsymbol{i} - 2\boldsymbol{j} + 6\boldsymbol{k}$

Solution

Let $\boldsymbol{a} = 3\boldsymbol{i} - 2\boldsymbol{j} + 6\boldsymbol{k}$

$$\|\boldsymbol{a}\| = \sqrt{(3)^2 + (-2)^2 + (6)^2} = 7$$

Determine the direction cosines

$$l = \cos\alpha = \frac{x}{\|\boldsymbol{a}\|} = \frac{1}{7}$$

$$m = \cos\beta = \frac{y}{\|\boldsymbol{a}\|} = -\frac{2}{7}$$

$$n = \cos\gamma = \frac{z}{\|\boldsymbol{a}\|} = \frac{6}{7}$$

Check

$$l^2 + m^2 + n^2 = \left(\frac{1}{7}\right)^2 + \left(-\frac{2}{7}\right)^2 + \left(\frac{6}{7}\right)^2 = 1$$

$[l, m, n] = \left[\frac{1}{7}, -\frac{2}{7}, \frac{6}{7}\right]$.