# Answer on Question #83335 – Math – Analytic Geometry

## Question

**(1)** If $r = 2i - 4j, s = 2i + 6j, t = 3i - j$, find $2r - t + s$

## Solution

$2r - t + s = 2(2i - 4j) - (3i - j) + 2i + 6j = 3i - j$

## Question

**(2)** Find the equation of the tangent to the curve $y = x^3 - x^2$ at the point $(1, 0)$.

## Solution

Find the first derivative with respect to $x$

$$y' = (x^3 - x^2)' = 3x^2 - 2x$$

Find the slope of the tangent line

$$slope = m = y'(1) = 3(1)^2 - 2(1) = 1$$

The equation of the tangent to the curve in point slope form

$$y - 0 = 1(x - 1)$$

The equation of the tangent to the curve in slope-intercept form

$$y = x - 1$$

## Question

**(3)** Find the area of a triangle with vertices (3, 1), (0, 0) and (1, 2).

## Solution

Let $A(3,1), O(0,0)$, and $B(1,2)$ be the vertices of a triangle.

$\overrightarrow{OA} = (x_A - x_O, y_A - y_O) = (3 - 0, 1 - 0) = (3, 1)$

$\overrightarrow{OB} = (x_B - x_O, y_B - y_O) = (1 - 0, 2 - 0) = (1, 2)$

Find the cross product

$$\overrightarrow{OA} \times \overrightarrow{OB} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 1 & 0 \\ 1 & 2 & 0 \end{vmatrix} = \vec{i}\begin{vmatrix} 1 & 0 \\ 2 & 0 \end{vmatrix} - \vec{j}\begin{vmatrix} 3 & 0 \\ 1 & 0 \end{vmatrix} + \vec{k}\begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix} = 0\vec{i} - 0\vec{j} + 5\vec{k}$$

$$S_{\Delta ABO} = \frac{1}{2}\left\|\overrightarrow{OA} \times \overrightarrow{OB}\right\| = \frac{1}{2}(5) = \frac{5}{2}\ (units^2)$$

## Question

**(4)** If the point $C(2, -1)$ be the centre of a circle that passes through the point $A(-2,2)$ find the equation of the circle.

**Solution**

The equation of the circle with center at $(h, k)$ and radius $r$

$$(x - h)^2 + (y - k)^2 = r^2$$

Substitute and find $r$

$$(-2 - 2)^2 + (2 - (-1))^2 = r^2$$

$$r^2 = 25 => r = 5$$

The equation of the circle with center at $C(2, -1)$ and radius 5

$$(x - 2)^2 + (y + 1)^2 = 25$$

**Question**

**(5)** If $r = 2i - 4j, s = 2i + 6j, t = 3i - j$, find magnitude of the vector $2r - t + s$

**Solution**

$2r - t + s = 2(2i - 4j) - (3i - j) + 2i + 6j = 3i - j$

Find magnitude of the vector $2r - t + s$

$$\|2r - t + s\| = \sqrt{(3)^2 + (-1)^2} = \sqrt{10}$$

**Question**

**(6)** $P, Q$ and $R$ are points $(1, -6), (3, 6)$ and $(5, 2)$ respectively. Determine the length of the line joining the midpoint of $PQ$ and $QR$.

**Solution**

The midpoint of two points

$$x_m = \frac{x_1 + x_2}{2}, y_m = \frac{y_1 + y_2}{2}$$

The midpoint of $PQ$

$$x_{mPQ} = \frac{1 + 3}{2} = 2, y_{mPQ} = \frac{-6 + 6}{2} = 0$$

The midpoint of $QR$

$$x_{mQR} = \frac{3 + 5}{2} = 4, y_{mQR} = \frac{6 + 2}{2} = 4$$

Determine the length of the line joining the midpoint of $PQ$ and $QR$.

$$l = \sqrt{(4 - 2)^2 + (4 - 0)^2} = 2\sqrt{5}$$

**Question**

**(7)** Find the equation of a straight line passing through the points $(2, 3)$ and $(-2, 5)$.

**Solution**

Gradient (slope) of a straight line

$$grad = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-3}{-2-2} = -\frac{1}{2}$$

The equation of the line

$$y = -\frac{1}{2}x + b$$

Substitute and find $b$

$$3 = -\frac{1}{2}(2) + b => b = 4$$

The equation of a straight line passing through the points $(2, 3)$ and $(-2, 5)$.

$$y = -\frac{1}{2}x + 4$$

**Question**

**(8)** Let $Z_1 = 6\vec{\imath} - 4\vec{\jmath} + 4\vec{k}, Z_2 = \vec{\imath} + 6\vec{\jmath} - \vec{k}$, find magnitude of the cross product $\overrightarrow{Z_1} \times \overrightarrow{Z_2}$.

**Solution**

Find the cross product $\overrightarrow{Z_1} \times \overrightarrow{Z_2}$

$$\overrightarrow{Z_1} \times \overrightarrow{Z_2} = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ 6 & -4 & 4 \\ 1 & 6 & -1 \end{vmatrix} = \vec{\imath}\begin{vmatrix} -4 & 4 \\ 6 & -1 \end{vmatrix} - \vec{\jmath}\begin{vmatrix} 6 & 4 \\ 1 & -1 \end{vmatrix} + \vec{k}\begin{vmatrix} 6 & -4 \\ 1 & 6 \end{vmatrix} =$$

$$= \vec{\imath}\big(-4(-1) - 4(6)\big) - \vec{\jmath}\big(6(-1) - 4(1)\big) + \vec{k}\big(6(6) - (-4)(1)\big) =$$

$$= -20\vec{\imath} + 10\vec{\jmath} + 40\vec{k}$$

Find magnitude of the cross product $\overrightarrow{Z_1} \times \overrightarrow{Z_2}$

$$\left\|\overrightarrow{Z_1} \times \overrightarrow{Z_2}\right\| = \sqrt{(-20)^2 + (10)^2 + (40)^2} = 10\sqrt{21}$$

**Question**

**(9)** Find the equation of the normal to the curve $y = x^3 - x^2$ at the point $(1, 1)$.

**Solution**

Find the first derivative with respect to $x$

$$y' = (x^3 - x^2)' = 3x^2 - 2x$$

Find the slope of the normal line

$$slope = m = -\frac{1}{y'(1)} = -\frac{1}{3(1)^2 - 2(1)} = -1$$

The equation of the normal to the curve in point slope form

$$y - 1 = -1(x - 1)$$

The equation of the normal to the curve in slope-intercept form

$$y = -x + 2$$