Answer on Question #83324 – Math – Calculus

Question

Find the maximum possible domain, and the corresponding range of the function f given by:

$$f(x) = \frac{1}{1 + cosx}$$

Solution

Let's find the domain, that is, values of x for which the function is not defined. The denominator of the fraction is zero.

$1 + cosx = 0$

$cosx = -1$

$x = \pi + 2\pi k, k \in Z$

In this case, x belongs to $(-\infty; +\infty)\backslash\{\pi + 2\pi k, k \in Z\}$

Let's find the range, the region in which y is defined.

$(f(x))' = \left(\frac{1}{1 + cosx}\right)' = \frac{sinx}{(1 + cosx)^2}$

The function reaches an extremum if the value of its derivative is zero.

$\frac{sinx}{(1 + cosx)^2} = 0$

$x = \pi t, t \in Z$

However, the denominator should not be equal to zero; therefore, only the values:

$x = 2\pi t, t \in Z$

Substitute the value of this point to find out the value of the function at the extremum.

$$f(0) = \frac{1}{1 + \cos(0)} = \frac{1}{2}$$

$$\lim_{x \to \pi k,\ k \in Z} f(x) = +\infty$$

In this case, y belongs to $[0.5, +\infty)$

Answer:

The domain of the function is $(-\infty, +\infty)\backslash\{\pi + 2\pi k, k \in Z\}$, and the range of the function is $[0.5, +\infty)$.