Answer to Question #83269 - Math - Calculus

Question:

Find the maximum and minimum values of the curve and distinguish them. y=x^3-6x^2+9x+6

Solution:

$y = x^3 - 6x^2 + 9x + 6$

$y' = 3x^2 - 12x + 9$

$y'' = 6x - 12$

To find critical points, take $y' = 0$

$3x^2 - 12x + 9 = 0$

$x^2 - 4x + 3 = 0$

$(x-1)(x-3) = 0$

The critical points are $x = 1$ and $x = 3$.

When $x = 1$, $y'' = 6(1) - 12 = -6 < 0$.

Hence the function has a local maximum at $x = 1$.

Local maximum value of the function = $y = 1^3 - 6(1)^2 + 9(1) + 6 = 1 - 6 + 9 + 6 = 10$.

When $x = 3$, $y'' = 6(3) - 12 = 6 > 0$.

Hence the function has a local minimum at $x = 3$.

Local minimum value of the function = $y = 3^3 - 6(3)^2 + 9(3) + 6 = 27 - 54 + 27 + 6 = 6$.

The Domain of the function $y = x^3 - 6x^2 + 9x + 6$ is $(-\infty, \infty)$.

x	$(-\infty, 1)$	1	$(1, 3)$	3	$(3, \infty)$
Sign of y'	Positive	0	Negative	0	Positive
Graph of y	Increasing	Maximum	Decreasing	Minimum	Increasing

Absolute maximum value of $y = \infty$ at $x = \infty$.

Absolute minimum value of $y = -\infty$ at $x = -\infty$.

Local maximum value of $y = 10$ at $x = 1$.

Local minimum value of $y = 6$ at $x = 3$.