Answer on Question #83225 - Math - Combinatorics - Number Theory

Question

In how many ways can a class of 22 students be split into two groups of 9 and 13 respectively if there is a pair of twins who must not be separated?

Solution

Consider two independent options.

1. Let both twins be in the first group, which contains 9 people. Then you can reformulate the task as how many ways you can break 20 people into two groups of 7 and 13 people. This can be calculated by the combination formula without repetitions.

Here x1 is the number of options in the first case:

$$x1 = C_{20}^{7} = \frac{20!}{7!\,13!} = \frac{20 * 19 * 18 * 17 * 16 * 15 * 14}{2 * 3 * 4 * 5 * 6 * 7} = 77520$$

2. Let both twins be in the second group, which contains 13 people. Then you can reformulate the task as how many ways you can break 20 people into two groups of 9 and 11 people. This can be calculated by the combination formula without repetitions.

Here x2 is the number of options in the second case:

$$x2 = C_{20}^{9} = \frac{20!}{9!\,11!} = \frac{20 * 19 * 18 * 17 * 16 * 15 * 14 * 13 * 12}{2 * 3 * 4 * 5 * 6 * 7 * 8 * 9} = 167960$$

According to the rule of addition, the final answer is equal to the sum of these two numbers, that is,

$$x = x1 + x2 = 77520 + 167960 = 245480$$

Answer: 245480 ways.