

Answer on Question #83224 – Math – Calculus

Question

Find the maximum and minimum values of the curve and distinguish them

$$y = x^3 - 6x^2 + 9x + 6$$

Solution

Let's take the derivative of the function. According to the rule, if the value of the derivative is greater than zero then the function will increase, and, if the derivative is less than zero then the function will decrease. Therefore, at the points where the derivative will be equal to 0, the function will have a relative extremum.

Compute the first derivative:

$$y' = (x^3 - 6x^2 + 9x + 6)' = 3x^2 - 12x + 9 = 3(x^2 - 4x + 3) = 3(x - 1)(x - 3)$$

Let's find out the value of the function at points $x = 1$ and $x = 3$:

$$y(1) = 1^3 - 6 * 1^2 + 9 * 1 + 6 = 1 - 6 + 9 + 6 = 10$$

$$y(3) = 3^3 - 6 * 3^2 + 9 * 3 + 6 = 27 - 36 + 18 + 6 = 6$$

The function increases on $x \leq 1$, decreases on $[1, 3]$ and increases on $x \geq 3$ again.

$$\lim_{x \rightarrow +\infty} (x^3 - 6x^2 + 9x + 6) = +\infty$$

$$\lim_{x \rightarrow -\infty} (x^3 - 6x^2 + 9x + 6) = -\infty$$

Answer:

The function takes on the smallest and largest values when it tends to minus infinity and plus infinity, respectively. However, it has a local maximum at $x = 1$, where the function value is 10, and a local minimum at $x = 3$, where the function value is 6.