

## Answer on Question #83220 – Math – Statistics and Probability

### Question

A shipment of ten similar plasma televisions contains 3 that are defective. If a hotel makes a random purchase of 2 of these television sets, find:

- The probability distribution for the number of defectives television sets.
- The probability that the hotel will purchase at least one defective television set.
- The mean the defective television sets.
- The variance of the defective television sets.

### Solution

Let  $x$  be the number of defective sets purchased by the hotel.

Assume  $X$  is a random variable that represents the number of defective sets purchased by the hotel.

The total number of television sets is  $N = 10$ .

The defective number of television sets is  $m = 3$ .

The number of selected televisions for purchase is  $n = 2$ .

The distribution of  $X$  follows the hypergeometric distribution with parameters  $N = 10, n = 2$  and  $m = 3$ .

a) The probability mass function for the hypergeometric distribution is as follows:

$$p(x, N, n, m) = Pr(X = x) = \frac{\binom{m}{x} \binom{N-m}{n-x}}{\binom{N}{n}}$$

$$Pr(X = x) = \frac{\binom{3}{x} \binom{10-3}{2-x}}{\binom{10}{2}}, x = 0, 1, 2$$

b) The probability that the hotel will purchase at least one defective television set

$$\begin{aligned} Pr(X \geq 1) &= 1 - Pr(X = 0) = 1 - \frac{\binom{3}{0} \binom{10-3}{2-0}}{\binom{10}{2}} = 1 - \frac{\frac{7!}{2!(7-2)!} (1)}{\frac{10!}{2!(10-2)!}} \\ &= 1 - \frac{21}{45} = \frac{8}{15} \end{aligned}$$

c) The mean the defective television sets

$$\mu = E[X] = np = n \left( \frac{m}{N} \right) = 2 \left( \frac{3}{10} \right) = \frac{3}{5}$$

d) The variance of the defective television sets.

$$\sigma^2 = Var[X] = \frac{np(1-p)(N-n)}{N-1} = \frac{n \left( \frac{m}{N} \right) \left( 1 - \frac{m}{N} \right) (N-n)}{N-1}$$

$$\sigma^2 = \text{Var}[X] = \frac{2 \left(\frac{3}{10}\right) \left(1 - \frac{3}{10}\right) (10 - 2)}{10 - 1} = \frac{28}{75}$$