Question

Given a function f, defined on R by $f(x) = x^2/(x^2 + 4)$, l = 1 and $\varepsilon = 0.1$, find $k > 0$ such that $x > k =>$ $|f(x) - 1| < \varepsilon$.

Solution

We have $f(x) - 1 = (x^2 - (x^2 + 4))/(x^2 + 4) = -4/(x^2 + 4)$, so that $|f(x) - 1| = 4/(x^2 + 4)$. The inequality $|f(x) - 1| < \varepsilon$ then reads $4/(x^2 + 4) < \varepsilon$ and has solution $x^2 > 4/\varepsilon - 4$. Substituting $\varepsilon = 0.1$, we get $x^2 > 36$, hence $|x| > 6$.
Answer: $k = 6$.