

Answer on Question #83136 – Math – Calculus

Question

3. Let f and h be functions, defined on \mathbb{R} by

$$f(x) = x^3 - 3x^2 - 4x + 12 \text{ and}$$

$$h(x) = \begin{cases} f(x)/(x - 3), & \text{for } x \text{ not equal to 3,} \\ k \text{ for } x = 3. \end{cases}$$

(i) Find all the roots of $f(x) = 0$.

(ii) Find the value of k that makes h continuous at $x = 3$.

(iii) Using the value of k found in (ii) above, determine if h is an even function or not.

Solution

(i) Note that $x = 3$ is a root of $f(x) = 0$. Using this, and factorizing $f(x)$, we have

$$f(x) = x^3 - 3x^2 - 4x + 12 = (x - 3)(x - 2)(x + 2).$$

Answer: the roots of $f(x) = 0$ are 3, 2, and -2.

(ii) Using the previous result, we have $h(x) = f(x)/(x - 3) = x^2 - 4$ for x not equal to 3. By continuity, this function extends to $x = 3$ with the result $h(3) = 3^2 - 4 = 5$.

Answer: $k = 5$.

(iii) The constructed continuous function $h(x) = x^2 - 4$ is obviously even.

Answer: it is even.