Question

Let W be a random variable giving the number of heads minus the number of tails in FOUR tosses of a coin. List the elements of the sample space S for the FOUR tosses of the coin and to each sample point assign a value w of W.

b) Find the probability distribution in tabular form.

c) Find the probability mass function for W.

Solution

The elements of the sample space S for the FOUR tosses of the coin are:

W = {-4, -2, 0, 2, 4}

b) The probability distribution in tabular form:

w	-4	-2	0	2	4
P(w)	1/16	4/16	6/16	4/16	1/16

Where **w** is a value of W, and *P(w)* is probability of *w*

c) The probability mass function for W:

$$P(w) = \binom{n}{k} p^k (1-p)^{n-k}$$

Where **p** is probability of heads in ONE toss of a coin, **p** = 1/2, *(1-p)* =1/2

$$\binom{n}{k} = \frac{n!}{k!(n-k)!} \; ,$$

$$k = \frac{w}{2} + 2 \; , \quad n = 4.$$

Thus, $P(w) = \binom{n}{k} p^k (1-p)^{n-k} = \dfrac{4!}{\left(\frac{w}{2}+2\right)!\left(4-\frac{w}{2}-2\right)!} * \left(\frac{1}{2}\right)^4 = \dfrac{24}{16*\left(2+\frac{w}{2}\right)!\left(2-\frac{w}{2}\right)!}$

Answer:

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