

Answer to Question #83127

Let us consider the map $f: \mathbb{Z} + x\mathbb{Q}[x] \rightarrow \mathbb{Z}_2$ which maps any polynomial into the residue modulo 2 of the last coefficient. This map is homomorphism as a composition of standard last coefficient homomorphism $\mathbb{Z} + x\mathbb{Q}[x] \rightarrow \mathbb{Z}$ and $\mathbb{Z} \rightarrow \mathbb{Z}_2$. $\text{Ker}(f) = \langle 2, x \rangle$. \mathbb{Z}_2 is a field. By the homomorphism theorem $(\mathbb{Z} + x\mathbb{Q}[x]) / \langle 2, x \rangle$ is isomorphic to \mathbb{Z}_2 , hence it is a field. So $\langle 2, x \rangle$ is by definition maximal.