Answer on Question #83090 – Math – Functional Analysis

Question

If $f(x) = f(y)$ for every bounded linear functional f on a normed space X, show that $x = y$.

Solution

It is false.

Let $f(x)$ be different from 0 bounded linear functional f, write its kernel:

$Ker\,f = \{x \in X: f(x) = 0\} \neq 0$ and take $x_0 \in Ker\,f$, $x_0 \neq 0$ (such an element x_0 always exists) then for elements $x + x_0$ and x we have $f(x + x_0) = f(x) + f(x_0) = f(x)$ but $x \neq x + x_0$.

Answer: It is false.