Let’s consider a particular case of Taylor Series, in the region near x = 0. Such a polynomial is called the Maclaurin Series. The infinite series expansion for $f(x)$ about x = 0 becomes:

$$f(x) \approx f(0) + f'(0)x + \frac{f''(0)}{2}x^2 + \frac{f'''(0)}{3!}x^3 + \frac{f^{\text{iv}}(0)}{4!}x^4 + \ldots$$

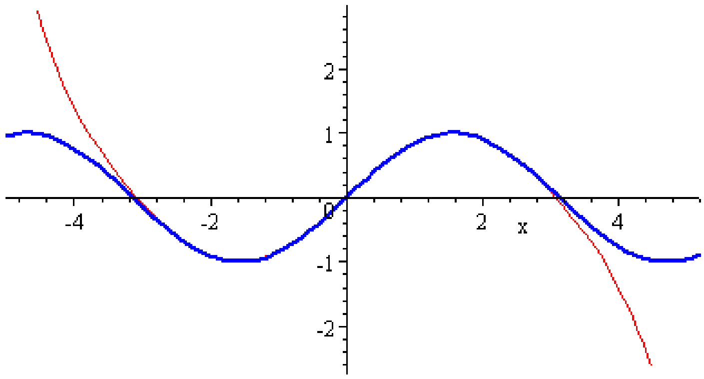
Let’s find the Maclaurin Series expansion for $f(x)$ = sin x:

$$sinx = \sin 0 + x\cos 0 + \frac{x^2 \sin 0}{2} + \frac{\mathrm{x}^3 \cos 0}{3!} + \cdots = x - \frac{1}{6}x^3 + \frac{1}{120}x^5 - \frac{1}{5040}x^7 + \cdots$$

and for $f(x)$ = cos x:

$$cosx = \cos 0 + x\sin 0 + \frac{x^2 \cos 0}{2} + \frac{\mathrm{x}^3 \sin 0}{3!} + \cdots = 1 - \frac{1}{2}x^3 + \frac{1}{24}x^4 - \frac{1}{720}x^6 + \cdots$$

We plot our answer to see if the polynomial is a good approximation to $f(x)$ = sin x.



We observe that our polynomial (in red) is a good approximation to $f(x)$ = sin x (in blue) near x = 0. In fact, it is quite good between $-3 \le x \le 3$. So, the statement that the Maclaurin series converge to their respective function for all x is wrong.