Answer on Question #83036 – Math – Analytic Geometry

Question

line AB passes through the point P(3,-2) with gradient -1/2, determine the equation of the line CD through P perpendicular to AB.

Solution

$$\text{AB: } y = \mathrm{k}_1\mathrm{x} + \mathrm{b}_1$$

$$\text{CD: } y = \mathrm{k}_2 x + \mathrm{b}_2$$

$$\text{CD: } \mathrm{y} - \mathrm{y}_0 = \mathrm{k}_2(\mathrm{x} - \mathrm{x}_0)$$

It is given that CD is perpendicular to AB => $\mathrm{k}_2 = -\frac{1}{\mathrm{k}_1}$,
where k_1 is the gradient of AB and k_2 is the gradient of CD;

$$\mathrm{k}_1 = -\frac{1}{2} \Rightarrow \mathrm{k}_2 = -\frac{1}{-\frac{1}{2}} = 2$$

Besides, CD goes through the point $\mathrm{P}(3, -2)$. Thus, $\mathrm{k}_2 = 2, x_0 = 3, y_0 = -2$.
Finally,

$$\text{CD: } \mathrm{y} - (-2) = 2(\mathrm{x} - 3)$$
$$\mathrm{y} + 2 = 2\mathrm{x} - 6$$
$$\mathrm{y} = 2\mathrm{x} - 6 - 2$$
$$\mathrm{y} = 2\mathrm{x} - 8$$

Answer: $\mathrm{y} = 2\mathrm{x} - 8.$