if f(x)=x^n, where n is an element of N, show that f'(a)=na^n-1 for any a.

To prove the power rule for differentiation, we use the definition of the derivative as a limit. But first, note the factorization for $n \geq 1$:

$$f(x) - f(a) = x^n - a^n = (x - a)(x^{n-1} + ax^{n-2} + \cdots + a^{n-2}x + a^{n-1})$$

Using this, we can see that

$$f'(a) = \lim_{x \to a} \frac{x^n - a^n}{x - a} = \lim_{x \to a} x^{n-1} + ax^{n-2} + \cdots + a^{n-2}x + a^{n-1}$$

Since the division has been eliminated and we have a continuous function, we can freely substitute to find the limit:

$$f'(a) = \lim_{x \to a} x^{n-1} + ax^{n-2} + \cdots + a^{n-2}x + a^{n-1} = a^{n-1} + a^{n-1} + \cdots + a^{n-1} + a^{n-1} = n \cdot a^{n-1}$$

The case of $n = 0$ is trivial because $x^0 = 1$, so

$$\frac{d}{dx} 1 = 0 = 0 \cdot x^{-1}.$$

The use of the quotient rule allows the extension of this rule for n as a negative integer, and the use of the laws of exponents and the chain rule allows this rule to be extended to all rational values of *n*. For an irrational n, a rational approximation is appropriate.