Answer on Question #82988 – Math – Combinatorics | Number Theory

Question 1

5 girls and 5 boys are to be seated around a circular table such that no 2 girls will sit together the number of ways in which they can be seated around the table is?

Solution

5 boys can sit around the circular table in: $(5-1)! = 4!$

For boys and girls to occupy alternate positions, 5 girls has to sit in the gap between the 5 boys.

The girls can be arranged in these gaps in $5!$ ways.

Answer: $N = 4! \cdot 5! = 2880$

Question 2

6 women and 5 men are to be seated in a row so that no two men can sit together number of ways they can be seated is?

Solution

In case of M W M W M W M W M W M:

6 women can sit in $6!$ ways and 5 men can sit in $5!$

$$N_1 = 6! \cdot 5! = 86400$$

In case of M W M W M W M W M W W:

$$N_2 = 6! \cdot 5! = 86400$$

In case of W W M W M W M W M W M:

$$N_3 = 6! \cdot 5! = 86400$$

Answer: $N = N_1 + N_2 + N_3 = 86400 + 84600 + 84600 = 259200$

Question 3

3. What is circular permutation and why we do $(n-1)!$ and $(n-1)!/2$?

Solution

A circular permutation is a type of permutation which has no starting point and no ending point. It is a set of elements that has an order, but no reference point. It circles back around on itself and encloses.

a) Clockwise and counter-clockwise orders are different.

Given a circular arrangement of *n* objects, they can be rotated $0,1,2,\ldots,n-1$ places clockwise without changing the relative order of the objects. Thus, the total number of circular permutations is

$$\frac{n!}{n}=(n-1)!$$

b) Clockwise and counter-clockwise orders are not different.

In this case, observation can be made from both sides, and this will be the same. Here two permutations will be counted as one. Thus, the number of circular permutations is

$$\frac{(n-1)!}{2}$$