ANSWER on Question #82982 – Math – Calculus

QUESTION

Definite integral:

Integral of uppercase square root of pi/2 and lower case 0 x e raise to 2x cos x squared dx

$$\int_0^{\sqrt{\pi/2}} x\cdot e^{2x}\cdot \cos^2 x\,dx$$

Or

$$\int_0^{\sqrt{\pi}/2} x\cdot e^{2x}\cdot \cos^2 x\,dx$$

SOLUTION

first we find the indefinite integral

$$\int x\cdot e^{2x}\cdot\cos^2 x\,dx=\int x\cdot e^{2x}\cdot\left(\frac{1+\cos 2x}{2}\right)dx=\frac{1}{2}\cdot\int x\cdot e^{2x}\cdot(1+\cos 2x)dx=$$

$$=\frac{1}{2}\cdot\int x\cdot e^{2x}dx+\frac{1}{2}\cdot\int x\cdot e^{2x}\cdot\cos(2x)\,dx=I_1+I_2$$

$$I_1=\frac{1}{2}\cdot\int x\cdot e^{2x}dx=\begin{bmatrix}u=x\to du=dx\\ dv=e^{2x}dx\to v=\frac{1}{2}\cdot e^{2x}\end{bmatrix}=\frac{1}{2}\cdot\left(x\cdot\frac{1}{2}\cdot e^{2x}-\int\frac{1}{2}\cdot e^{2x}dx\right)=$$

$$=\frac{x\cdot e^{2x}}{4}-\frac{1}{4}\cdot\frac{1}{2}\cdot e^{2x}+C=\frac{x\cdot e^{2x}}{4}-\frac{e^{2x}}{8}+C=\frac{e^{2x}}{8}\cdot(2x-1)+C$$

$$\boxed{I_1=\frac{1}{2}\cdot\int x\cdot e^{2x}dx=\frac{e^{2x}}{8}\cdot(2x-1)+C}$$

$$I_2=\frac{1}{2}\cdot\int x\cdot e^{2x}\cdot\cos(2x)\,dx=\begin{bmatrix}2x=t\to x=\frac{t}{2}\\ dx=\frac{dt}{2}\end{bmatrix}=\frac{1}{2}\cdot\int\frac{t}{2}\cdot e^{t}\cdot\cos t\,\frac{dt}{2}=$$

$$=\frac{1}{8}\cdot\int t\cdot e^{t}\cdot\cos t\,dt=$$

Now we'll use two formulas:

$$\int e^t \cdot \cos t \, dt = \frac{e^t}{2} \cdot (\sin t + \cos t)$$

$$\int e^t \cdot \sin t \, dt = \frac{e^t}{2} \cdot (\sin t - \cos t)$$

Then,

$$I_2 = \frac{1}{8} \cdot \int t \cdot e^t \cdot \cos t \, dt = \begin{bmatrix} u = t \to du = dt \\ dv = e^t \cdot \cos t \, dt \to v = \frac{e^t}{2} \cdot (\sin t + \cos t) \end{bmatrix} =$$

$$= \frac{1}{8} \cdot \left(\frac{t \cdot e^t}{2} \cdot (\sin t + \cos t) - \int \frac{e^t}{2} \cdot (\sin t + \cos t) dt \right) =$$

$$= \frac{1}{16} \cdot \left(te^t (\sin t + \cos t) - \int e^t \sin t \, dt - \int e^t \cos t \, dt \right) =$$

$$= \frac{1}{16} \cdot \left(te^t (\sin t + \cos t) - \frac{e^t}{2} \cdot (\sin t - \cos t) - \frac{e^t}{2} \cdot (\sin t + \cos t) \right) + C =$$

$$= \frac{1}{16} \cdot \left(te^t (\sin t + \cos t) - \frac{e^t}{2} \cdot (\sin t - \cos t + \sin t + \cos t) \right) + C =$$

$$= \frac{1}{16} \cdot \left(te^t (\sin t + \cos t) - \frac{e^t}{2} \cdot (2 \sin t) \right) = \frac{1}{16} \cdot \left(te^t (\sin t + \cos t) - e^t \sin t \right) + C =$$

$$= \frac{e^t}{16} \cdot \big((t-1) \cdot \sin t + t \cdot \cos t \big) + C$$

$$\boxed{I_2 = \frac{1}{2} \cdot \int x \cdot e^{2x} \cdot \cos(2x) \, dx = \frac{e^{2x}}{16} \cdot \big((2x-1) \cdot \sin(2x) + 2x \cdot \cos(2x) \big) + C}$$

Conclusion,

$$\boxed{\int x \cdot e^{2x} \cdot \cos^2 x \, dx = I_1 + I_2 = \frac{e^{2x}}{8} \cdot (2x-1) + \frac{e^{2x}}{16} \cdot \big((2x-1) \cdot \sin(2x) + 2x \cdot \cos(2x) \big) + C}$$

Then,

$$\boxed{\int_0^{\sqrt{\pi/2}} x \cdot e^{2x} \cdot \cos^2 x \, dx = \left(\frac{e^{2x}}{8} \cdot (2x-1) + \frac{e^{2x}}{16} \cdot \left((2x-1) \cdot \sin(2x) + 2x \cdot \cos(2x)\right)\right)\Bigg|_0^{\sqrt{\pi/2}} \approx 1.5727}$$

and

$$\int_0^{\sqrt{\pi}/2} x \cdot e^{2x} \cdot \cos^2 x \, dx = \left(\frac{e^{2x}}{8} \cdot (2x-1) + \frac{e^{2x}}{16} \cdot \left((2x-1) \cdot \sin(2x) + 2x \cdot \cos(2x)\right)\right)\Bigg|_0^{\sqrt{\pi}/2} =$$

$$= \left(\frac{e^{\sqrt{\pi}}}{8} \cdot \left(\sqrt{\pi}-1\right) + \frac{e^{\sqrt{\pi}}}{16} \cdot \left(\left(\sqrt{\pi}-1\right) \cdot \sin\left(\sqrt{\pi}\right) + \sqrt{\pi} \cdot \cos\left(\sqrt{\pi}\right)\right)\right) -$$

$$-\left(\frac{e^0}{8} \cdot (0-1) + \frac{e^0}{16} \cdot \left((0-1) \cdot \sin(0) + 0 \cdot \cos(0)\right)\right) =$$

$$= \frac{e^{\sqrt{\pi}}}{16} \cdot \left(2 \cdot \left(\sqrt{\pi}-1\right) + \left(\sqrt{\pi}-1\right) \cdot \sin\left(\sqrt{\pi}\right) + \sqrt{\pi} \cdot \cos\left(\sqrt{\pi}\right)\right) - \left(-\frac{1}{8}\right) =$$

$$= \frac{e^{\sqrt{\pi}}}{16} \cdot \left(2 \cdot \left(\sqrt{\pi}-1\right) + \left(\sqrt{\pi}-1\right) \cdot \sin\left(\sqrt{\pi}\right) + \sqrt{\pi} \cdot \cos\left(\sqrt{\pi}\right)\right) + \frac{2}{16} =$$

$$= \frac{1}{16} \cdot \left(2 + e^{\sqrt{\pi}} \cdot \left[\sqrt{\pi} \cdot \cos\left(\sqrt{\pi}\right) + \left(\sqrt{\pi}-1\right) \cdot \left(2 + \sin\left(\sqrt{\pi}\right)\right)\right]\right) \approx 0.8411$$

Conclusion,

$$\boxed{\int_0^{\sqrt{\pi}/2} x \cdot e^{2x} \cdot \cos^2 x \, dx = \frac{1}{16} \cdot \left(2 + e^{\sqrt{\pi}} \cdot \left[\sqrt{\pi} \cdot \cos\left(\sqrt{\pi}\right) + \left(\sqrt{\pi}-1\right) \cdot \left(2 + \sin\left(\sqrt{\pi}\right)\right)\right]\right) \approx 0.8411}$$

ANSWER:

$$\int_0^{\sqrt{\frac{\pi}{2}}} x\cdot e^{2x}\cdot \cos^2 x\,dx = \left(\frac{e^{2x}}{8}\cdot(2x-1)+\frac{e^{2x}}{16}\cdot\big((2x-1)\cdot\sin(2x)+2x\cdot\cos(2x)\big)\right)\Bigg|_0^{\sqrt{\frac{\pi}{2}}} \approx 1.5727$$

Or

$$\int_0^{\sqrt{\pi}/2} x\cdot e^{2x}\cdot \cos^2 x\,dx = \frac{1}{16}\cdot\left(2+e^{\sqrt{\pi}}\cdot\left[\sqrt{\pi}\cdot\cos(\sqrt{\pi})+(\sqrt{\pi}-1)\cdot(2+\sin(\sqrt{\pi}))\right]\right) \approx 0.8411$$