Question

d^2u/dx^2 =6x+12y^2 subject to u(1,y) = y^2-2y ,u(x,t)=5x-5

Solution

$$\frac{d^2u(x,y)}{dx^2} = 6x + 12y^2$$

$$\frac{du(x,y)}{dx} = \int 6x + 12y^2 \, dx = 3(2y^2 + x)^2 + C1(y)$$

$$u(x,y) = \int \left(3(2y^2 + x)^2 + C1(y)\right) dx = (2y^2 + x)^3 + x \cdot C1(y) + C2(y)$$

$$u(1,y) = y^2 - 2y \rightarrow C2(y) = y^2 - 2y - (2y^2 + 1)^3 - C1(y)$$

$$u(x,y) = (2y^2 + x)^3 + (x - 1)C1(y) + y^2 - 2y - (2y^2 + 1)^3$$

$$u(x,t) = 5x - 5 \rightarrow u(x,t) = (2t^2 + x)^3 + (x - 1)C1(t) + t^2 - 2t - (2t^2 + 1)^3 = 5x - 5$$
$$\rightarrow C1(y) = 5$$

$$u(x,y) = (2y^2 + x)^3 + 5(x - 1) + y^2 - 2y - (2y^2 + 1)^3$$

Answer: $u(x,y) = (2y^2 + x)^3 + 5(x - 1) + y^2 - 2y - (2y^2 + 1)^3.$