Answer on Question #82971 – Math – Calculus

Question

1. $\int_{1/4}^{1/2} \text{Arcsin}\, x\, dx$

Solution

We integrate in parts:

$$\int_{\frac{1}{4}}^{\frac{1}{2}} \text{Arcsin}\, x\, dx = [x \cdot \text{Arcsin}\, x\, dx]_{\frac{1}{4}}^{\frac{1}{2}} - \int_{\frac{1}{4}}^{\frac{1}{2}} x\, d(\text{Arcsin}\, x) =$$

$$= [x \cdot \text{Arcsin}\, x]_{\frac{1}{4}}^{\frac{1}{2}} - \int_{\frac{1}{4}}^{\frac{1}{2}} \frac{x}{\sqrt{1-x^2}}\, dx = [x \cdot \text{Arcsin}\, x]_{\frac{1}{4}}^{\frac{1}{2}} + \left[\sqrt{1-x^2}\right]_{\frac{1}{4}}^{\frac{1}{2}} =$$

$$= \frac{1}{2} \cdot \text{Arcsin}\, \frac{1}{2} - \frac{1}{4} \cdot \text{Arcsin}\, \frac{1}{4} + \sqrt{1-\left(\frac{1}{2}\right)^2} - \sqrt{1-\left(\frac{1}{4}\right)^2} =$$

$$= \frac{1}{2} \cdot \frac{\pi}{6} - \frac{1}{4} \cdot \text{Arcsin}\, \frac{1}{4} + \sqrt{1-\frac{1}{4}} - \sqrt{1-\frac{1}{16}} = \frac{\pi}{12} - \frac{1}{4} \cdot \text{Arcsin}\, \frac{1}{4} + \frac{\sqrt{3}}{2} - \frac{\sqrt{15}}{4} \approx 0.096$$

Question

2. $\int_0^{\sqrt{\pi}/2} x e^{2x} \cos^2 x\, dx$

Solution

$$\int_0^{\frac{\sqrt{\pi}}{2}} x e^{2x} \cos^2 x\, dx = \int_0^{\frac{\sqrt{\pi}}{2}} x e^{2x} \frac{1+\cos 2x}{2} dx = \frac{1}{2}\int_0^{\frac{\sqrt{\pi}}{2}} x e^{2x} dx + \frac{1}{2}\int_0^{\frac{\sqrt{\pi}}{2}} x e^{2x} \cos 2x\, dx =$$

$$= \frac{1}{8}\int_0^{\frac{\sqrt{\pi}}{2}} 2x e^{2x} d2x + \frac{1}{8}\int_0^{\frac{\sqrt{\pi}}{2}} 2x e^{2x} \cos 2x\, d2x = \frac{1}{8}\int_0^{\sqrt{\pi}} y e^y dy + \frac{1}{8}\int_0^{\sqrt{\pi}} y e^y \cos y\, dy|_{y=2x} =$$

$$= \frac{1}{8}[e^y(y-1)]_0^{\sqrt{\pi}} + \frac{1}{16}[e^y(y\cos y + (y-1)\sin y)]_0^{\sqrt{\pi}} =$$

$$= \frac{1}{8} e^{\sqrt{\pi}}(\sqrt{\pi}-1) - \frac{1}{8} e^0(0-1) + \frac{1}{16} e^{\sqrt{\pi}}\left(\sqrt{\pi}\cos\sqrt{\pi} + (\sqrt{\pi}-1)\sin\sqrt{\pi}\right) -$$

$$- \frac{1}{16} e^0(0\cos 0 + (0-1)\sin 0) =$$

$$= \frac{1}{8} e^{\sqrt{\pi}}(\sqrt{\pi}-1) + \frac{1}{8} + \frac{1}{16} e^{\sqrt{\pi}}\left(\sqrt{\pi}\cos\sqrt{\pi} + (\sqrt{\pi}-1)\sin\sqrt{\pi}\right) \approx 0.841.$$