Answer on Question # 82965, Math / Calculus

$\frac{1}{1+x^2} < \arctan x < x$ doesn't hold. For example for $x = 0.1$ we have $\frac{1}{1+0.1^2} = 1 - \epsilon_1$ but $\arctan 0.1 = 0 + \epsilon_2$, where ϵ_1 and ϵ_2 in the neighbourhood of zero. I think that meant the following inequality:

correct question 1.

$$\forall x > 0\colon \frac{x}{1+x^2} < \arctan x < x$$

Proof. Note that $\arctan' x = \dfrac{1}{1+x^2}$.

Mean value theorem $\Rightarrow \exists a \in (0,x)\colon \dfrac{\arctan x - \arctan 0}{x-0} = \dfrac{1}{1+a^2}$.

Then $\dfrac{1}{1+x^2} < \dfrac{1}{1+a^2} < 1$ because $a \in (0,x)$.

So $\dfrac{1}{1+x^2} < \dfrac{\arctan x}{x} < 1$ and by x-multiplying $\dfrac{x}{1+x^2} < \arctan x < x$ for all $x > 0$. $\square$