ANSWER on Question #82963 – Math – Calculus

QUESTION

Definite integral:

1. Integral of uppercase 1/2 and lower case 1/4. Arcsin x dx

$$\int_{1/4}^{1/2} \arcsin x \, dx$$

2. Integral of uppercase 1/2 and lower case 1/4. Arctan x dx

$$\int_{1/4}^{1/2} \arctan x \, dx$$

3. Integral of uppercase square root of pi/2 and lower case 0 x e raise to 2x cos x squared dx

$$\int_{0}^{\sqrt{\pi/2}} x \cdot e^{2x} \cdot \cos^2 x \, dx$$

4. Integral of uppercase pi/2 and lower case 0. x e raise to 2x cos x squared dx

$$\int_{0}^{\pi/2} x \cdot e^{2x} \cdot \cos^2 x \, dx$$

SOLUTION

1.

$$\int_{1/4}^{1/2} \arcsin x \, dx = \left[\begin{matrix} u = \arcsin x \to du = \dfrac{dx}{\sqrt{1-x^2}} \\ dv = dx \to v = x \end{matrix}\right] = (x \cdot \arcsin x)\Big|_{1/4}^{1/2} - \int_{1/4}^{1/2} \frac{x \cdot dx}{\sqrt{1-x^2}} =$$

$$= \left(\frac{1}{2} \cdot \arcsin\left(\frac{1}{2}\right) - \frac{1}{4} \cdot \arcsin\left(\frac{1}{4}\right)\right) - \int_{1/4}^{1/2} \frac{1}{2} \cdot \frac{d(x^2)}{\sqrt{1-x^2}} = \left(\frac{1}{2} \cdot \arcsin\left(\frac{1}{2}\right) - \frac{1}{4} \cdot \arcsin\left(\frac{1}{4}\right)\right) - \left(\sqrt{1-x^2}\right)\Big|_{1/4}^{1/2} =$$

$$= \left(\frac{1}{2}\cdot \arcsin\left(\frac{1}{2}\right) - \frac{1}{4}\cdot \arcsin\left(\frac{1}{4}\right)\right) - \left(\sqrt{1-\frac{1}{4}} - \sqrt{1-\frac{1}{16}}\right) =$$

$$= \left(\frac{1}{2}\cdot \arcsin\left(\frac{1}{2}\right) - \frac{1}{4}\cdot \arcsin\left(\frac{1}{4}\right)\right) - \left(\frac{\sqrt{3}}{2} - \frac{\sqrt{15}}{4}\right) = \frac{1}{4}\cdot\left(2\cdot \arcsin\left(\frac{1}{2}\right) - \arcsin\left(\frac{1}{4}\right) - 2\sqrt{3} + \sqrt{15}\right)$$

Conclusion,

$$\boxed{\int_{1/4}^{1/2} \arcsin x\, dx = \frac{1}{4}\cdot\left(2\cdot \arcsin\left(\frac{1}{2}\right) - \arcsin\left(\frac{1}{4}\right) - 2\sqrt{3} + \sqrt{15}\right) \approx 0.3008}$$

2.

$$\int_{1/4}^{1/2} \arctan x\, dx = \left[\begin{matrix} u = \arctan x \to du = \dfrac{dx}{1+x^2} \\ dv = dx \to v = x \end{matrix}\right] = (x\cdot \arctan x)\Big|_{1/4}^{1/2} - \int_{1/4}^{1/2} \frac{x\cdot dx}{1+x^2} =$$

$$= \left(\frac{1}{2}\cdot \arctan\left(\frac{1}{2}\right) - \frac{1}{4}\cdot \arctan\left(\frac{1}{4}\right)\right) - \int_{1/4}^{1/2} \frac{1}{2}\cdot\frac{d(x^2)}{1+x^2} =$$

$$= \left(\frac{1}{2}\cdot \arctan\left(\frac{1}{2}\right) - \frac{1}{4}\cdot \arctan\left(\frac{1}{4}\right)\right) - \frac{1}{2}\cdot(\ln|1+x^2|)\Big|_{1/4}^{1/2} =$$

$$= \left(\frac{1}{2}\cdot \arctan\left(\frac{1}{2}\right) - \frac{1}{4}\cdot \arctan\left(\frac{1}{4}\right)\right) - \frac{1}{2}\cdot\left(\ln\left|1+\frac{1}{4}\right| - \ln\left|1+\frac{1}{16}\right|\right) =$$

$$= \left(\frac{1}{2}\cdot \arctan\left(\frac{1}{2}\right) - \frac{1}{4}\cdot \arctan\left(\frac{1}{4}\right)\right) - \frac{1}{2}\cdot\left(\ln\left|\frac{5}{4}\right| - \ln\left|\frac{17}{16}\right|\right) =$$

$$= \frac{1}{4}\cdot\left(2\cdot \arctan\left(\frac{1}{2}\right) - \arctan\left(\frac{1}{4}\right) - 2\cdot\ln\left|\frac{5}{4}\div\frac{17}{16}\right|\right) = \frac{1}{4}\cdot\left(2\cdot \arcsin\left(\frac{1}{2}\right) - \arcsin\left(\frac{1}{4}\right) - 2\cdot\ln\left|\frac{5}{4}\cdot\frac{16}{17}\right|\right) =$$

$$= \frac{1}{4}\cdot\left(2\cdot \arctan\left(\frac{1}{2}\right) - \arctan\left(\frac{1}{4}\right) - 2\cdot\ln\left|\frac{20}{17}\right|\right)$$

Conclusion,

$$\boxed{\int_{1/4}^{1/2} \arctan x\, dx = \frac{1}{4}\cdot\left(2\cdot \arctan\left(\frac{1}{2}\right) - \arctan\left(\frac{1}{4}\right) - 2\cdot\ln\left|\frac{20}{17}\right|\right) \approx 0.0891}$$

3, 4.

For part (3) and (4), first we find the indefinite integral

$$\int x\cdot e^{2x}\cdot\cos^2 x\,dx=\int x\cdot e^{2x}\cdot\left(\frac{1+\cos 2x}{2}\right)dx=\frac{1}{2}\cdot\int x\cdot e^{2x}\cdot(1+\cos 2x)dx=$$

$$=\frac{1}{2}\cdot\int x\cdot e^{2x}dx+\frac{1}{2}\cdot\int x\cdot e^{2x}\cdot\cos(2x)\,dx=I_1+I_2$$

$$I_1=\frac{1}{2}\cdot\int x\cdot e^{2x}dx=\begin{bmatrix}u=x\to du=dx\\ dv=e^{2x}dx\to v=\frac{1}{2}\cdot e^{2x}\end{bmatrix}=\frac{1}{2}\cdot\left(x\cdot\frac{1}{2}\cdot e^{2x}-\int\frac{1}{2}\cdot e^{2x}dx\right)=$$

$$=\frac{x\cdot e^{2x}}{4}-\frac{1}{4}\cdot\frac{1}{2}\cdot e^{2x}+C=\frac{x\cdot e^{2x}}{4}-\frac{e^{2x}}{8}+C=\frac{e^{2x}}{8}\cdot(2x-1)+C$$

$$\boxed{I_1=\frac{1}{2}\cdot\int x\cdot e^{2x}dx=\frac{e^{2x}}{8}\cdot(2x-1)+C}$$

$$I_2=\frac{1}{2}\cdot\int x\cdot e^{2x}\cdot\cos(2x)\,dx=\begin{bmatrix}2x=t\to x=\frac{t}{2}\\ dx=\frac{dt}{2}\end{bmatrix}=\frac{1}{2}\cdot\int\frac{t}{2}\cdot e^t\cdot\cos t\frac{dt}{2}=$$

$$=\frac{1}{8}\cdot\int t\cdot e^t\cdot\cos t\,dt=$$

Now we'll use two formulas:

$$\int e^t\cdot\cos t\,dt=\frac{e^t}{2}\cdot(\sin t+\cos t)$$

$$\int e^t\cdot\sin t\,dt=\frac{e^t}{2}\cdot(\sin t-\cos t)$$

Then,

$$I_2=\frac{1}{8}\cdot\int t\cdot e^t\cdot\cos t\,dt=\begin{bmatrix}u=t\to du=dt\\ dv=e^t\cdot\cos t\,dt\to v=\frac{e^t}{2}\cdot(\sin t+\cos t)\end{bmatrix}=$$

$$=\frac{1}{8}\cdot\left(\frac{t\cdot e^t}{2}\cdot(\sin t+\cos t)-\int\frac{e^t}{2}\cdot(\sin t+\cos t)dt\right)=$$

$$=\frac{1}{16}\cdot\left(te^t(\sin t+\cos t)-\int e^t\sin t\,dt-\int e^t\cos t\,dt\right)=$$

$$=\frac{1}{16}\cdot\left(te^t(\sin t+\cos t)-\frac{e^t}{2}\cdot(\sin t-\cos t)-\frac{e^t}{2}\cdot(\sin t+\cos t)\right)+C=$$

$$=\frac{1}{16}\cdot\left(te^t(\sin t+\cos t)-\frac{e^t}{2}\cdot(\sin t-\cos t+\sin t+\cos t)\right)+C=$$

$$=\frac{1}{16}\cdot\left(te^t(\sin t+\cos t)-\frac{e^t}{2}\cdot(2\sin t)\right)=\frac{1}{16}\cdot(te^t(\sin t+\cos t)-e^t\sin t)+C=$$

$$=\frac{e^t}{16}\cdot\big((t-1)\cdot\sin t+t\cdot\cos t\big)+C$$

$$\boxed{I_2=\frac{1}{2}\cdot\int x\cdot e^{2x}\cdot\cos(2x)\,dx=\frac{e^{2x}}{16}\cdot\big((2x-1)\cdot\sin(2x)+2x\cdot\cos(2x)\big)+C}$$

Conclusion,

$$\boxed{\int x\cdot e^{2x}\cdot\cos^2 x\,dx=I_1+I_2=\frac{e^{2x}}{8}\cdot(2x-1)+\frac{e^{2x}}{16}\cdot\big((2x-1)\cdot\sin(2x)+2x\cdot\cos(2x)\big)+C}$$

3.

$$\boxed{\int_0^{\sqrt{\pi/2}} x\cdot e^{2x}\cdot\cos^2 x\,dx=\left(\frac{e^{2x}}{8}\cdot(2x-1)+\frac{e^{2x}}{16}\cdot(2x\cdot\sin(2x)+(2x-1)\cdot\cos(2x))\right)\Bigg|_0^{\sqrt{\pi/2}}\approx 1.5727}$$

4.

$$\int_0^{\frac{\pi}{2}} x\cdot e^{2x}\cdot\cos^2 x\,dx=\left(\frac{e^{2x}}{8}\cdot(2x-1)+\frac{e^{2x}}{16}\cdot\big((2x-1)\cdot\sin(2x)+2x\cdot\cos(2x)\big)\right)\Bigg|_0^{\frac{\pi}{2}}$$

$$=\left(\frac{e^{\pi}}{8}\cdot(\pi-1)+\frac{e^{\pi}}{16}\cdot\big((\pi-1)\cdot\sin(\pi)+\pi\cdot\cos(\pi)\big)\right)$$

$$-\left(\frac{e^{0}}{8}\cdot(0-1)+\frac{e^{0}}{16}\cdot\big((0-1)\cdot\sin(0)+0\cdot\cos(0)\big)\right)=$$

$$=\left(\frac{e^{\pi}}{8}\cdot(\pi-1)-\frac{e^{\pi}}{16}\cdot\pi\right)-\left(-\frac{1}{8}+0\right)=\frac{e^{\pi}\cdot(2\pi-2-\pi)}{16}+\frac{1}{8}=\frac{1}{16}\cdot\big(2+e^{\pi}\cdot(\pi-2)\big)\approx 1.7761$$

$$\boxed{\int_0^{\frac{\pi}{2}} x \cdot e^{2x} \cdot \cos^2 x \, dx = \frac{1}{16} \cdot \left(2 + e^{\pi} \cdot (\pi - 2)\right) \approx 1.7761}$$

ANSWER:

1.

$$\int_{1/4}^{1/2} \arcsin x \, dx = \frac{1}{4} \cdot \left(2 \cdot \arcsin\left(\frac{1}{2}\right) - \arcsin\left(\frac{1}{4}\right) - 2\sqrt{3} + \sqrt{15}\right) \approx 0.3008$$

2.

$$\int_{1/4}^{1/2} \arctan x \, dx = \frac{1}{4} \cdot \left(2 \cdot \arctan\left(\frac{1}{2}\right) - \arctan\left(\frac{1}{4}\right) - 2 \cdot \ln\left|\frac{20}{17}\right|\right) \approx 0.0891$$

3.

$$\int_0^{\sqrt{\pi/2}} x \cdot e^{2x} \cdot \cos^2 x \, dx = \left(\frac{e^{2x}}{8} \cdot (2x - 1) + \frac{e^{2x}}{16} \cdot (2x \cdot \sin(2x) + (2x - 1) \cdot \cos(2x))\right)\Bigg|_0^{\sqrt{\pi/2}} \approx 1.5727$$

4.

$$\int_0^{\frac{\pi}{2}} x \cdot e^{2x} \cdot \cos^2 x \, dx = \frac{1}{16} \cdot \left(2 + e^{\pi} \cdot (\pi - 2)\right) \approx 1.7761$$