Answer on Question #82906 - Math - Linear Algebra

To solve an equation of the form $\mathbf{A}\,\vec{b}=\vec{u}$ where $\mathbf{A}$ is a matrix $\begin{bmatrix} 2 & 1 & -1 \\ 1 & -2 & 2 \\ 3 & -1 & 3 \end{bmatrix}$

$\vec{b}$ is a vector with unknown variables $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $\vec{u}$ is $\begin{bmatrix} 11 \\ -2 \\ 5 \end{bmatrix}$

we multiply both sides of the equation (from the right side) by $\mathbf{A}^{-1}$ (inverse of of $\mathbf{A}$ $\mathbf{A}\,\mathbf{A}^{-1}=\mathbf{A}^{-1}\,\mathbf{A}=\mathbf{I}$) obtained by formula $\mathbf{A}^{-1}=\frac{1}{|A|}Adj(A)$

where $|A|$ is determinant of A

$$|A|=2\left(\begin{vmatrix} -2 & 2 \\ -1 & 3 \end{vmatrix}\right)-\left(\begin{vmatrix} 1 & 2 \\ 3 & 3 \end{vmatrix}\right)-\left(\begin{vmatrix} 1 & -2 \\ 3 & -1 \end{vmatrix}\right)=-10$$

Adj(A) is adjugate of A and it’s obtained as follows :

Step 1 . Matrix of minors of A

$$\begin{bmatrix} \begin{vmatrix} -2 & 2 \\ -1 & 3 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 3 & 3 \end{vmatrix} & \begin{vmatrix} 1 & -2 \\ 3 & -1 \end{vmatrix} \\ \begin{vmatrix} 1 & -1 \\ -1 & 3 \end{vmatrix} & \begin{vmatrix} 2 & -1 \\ 3 & 3 \end{vmatrix} & \begin{vmatrix} 2 & 1 \\ 3 & -1 \end{vmatrix} \\ \begin{vmatrix} 1 & -1 \\ -2 & 2 \end{vmatrix} & \begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix} & \begin{vmatrix} 2 & 1 \\ 1 & -2 \end{vmatrix} \end{bmatrix}=\begin{bmatrix} -4 & -3 & 5 \\ 2 & 9 & -5 \\ 0 & 5 & -5 \end{bmatrix}$$

Step 2 . Change signs (multiply position-wise !)

$$\begin{bmatrix} -4 & -3 & 5 \\ 2 & 9 & -5 \\ 0 & 5 & -5 \end{bmatrix}\begin{bmatrix} +1 & -1 & +1 \\ -1 & +1 & -1 \\ +1 & -1 & +1 \end{bmatrix}=\begin{bmatrix} -4 & 3 & 5 \\ -2 & 9 & 5 \\ 0 & -5 & -5 \end{bmatrix}$$

Step 3 . Transpose

$$\begin{bmatrix} -4 & 3 & 5 \\ -2 & 9 & 5 \\ 0 & -5 & -5 \end{bmatrix}^T=\begin{bmatrix} -4 & -2 & 0 \\ 3 & 9 & -5 \\ 5 & 5 & -5 \end{bmatrix}$$

$$\mathbf{A}^{-1} = \frac{1}{-10}\begin{bmatrix} -4 & -2 & 0 \\ 3 & 9 & -5 \\ 5 & 5 & -5 \end{bmatrix} = \begin{bmatrix} 0.4 & 0.2 & 0 \\ -0.3 & -0.9 & .5 \\ -0.5 & -0.5 & 0.5 \end{bmatrix}$$

$$\mathbf{A}^{-1}\mathbf{A}\vec{\boldsymbol{b}} = \mathbf{A}^{-1}\vec{\boldsymbol{u}}$$

$$\begin{bmatrix} \mathbf{0.4} & \mathbf{0.2} & \mathbf{0} \\ -\mathbf{0.3} & -\mathbf{0.9} & \mathbf{.5} \\ -\mathbf{0.5} & -\mathbf{0.5} & \mathbf{0.5} \end{bmatrix}\begin{bmatrix} \mathbf{2} & \mathbf{1} & -\mathbf{1} \\ \mathbf{1} & -\mathbf{2} & \mathbf{2} \\ \mathbf{3} & -\mathbf{1} & \mathbf{3} \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \mathbf{0.4} & \mathbf{0.2} & \mathbf{0} \\ -\mathbf{0.3} & -\mathbf{0.9} & \mathbf{.5} \\ -\mathbf{0.5} & -\mathbf{0.5} & \mathbf{0.5} \end{bmatrix}\begin{bmatrix} \mathbf{11} \\ -\mathbf{2} \\ \mathbf{5} \end{bmatrix}$$

by applying inverse matrix we obtain solution : $\begin{bmatrix} \mathbf{4} \\ \mathbf{1} \\ -\mathbf{2} \end{bmatrix}$