

Answer on Question #82899 – Math – Linear Algebra

Question

solve the following equation using matrix algebra

$$2x+y-z=11$$

$$x-2y+2z=2$$

$$3x-y+3z=5$$

Solution

$$\begin{pmatrix} 2 & 1 & -1 \\ 1 & -2 & 2 \\ 3 & -1 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 11 \\ 2 \\ 5 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 & 1 & -1 \\ 1 & -2 & 2 \\ 3 & -1 & 3 \end{pmatrix}^{-1} \begin{pmatrix} 11 \\ 2 \\ 5 \end{pmatrix}$$

$$\begin{vmatrix} 2 & 1 & -1 \\ 1 & -2 & 2 \\ 3 & -1 & 3 \end{vmatrix} = 2 \cdot (-2) \cdot 3 + 1 \cdot 2 \cdot 3 + (-1) \cdot 1 \cdot (-1) - (-1) \cdot (-2) \cdot 3 - 2 \cdot 2 \cdot (-1) - 1 \cdot 1 \cdot 3 \\ = -12 + 6 + 1 - 6 + 4 - 3 = -10$$

$$\begin{pmatrix} 2 & 1 & -1 \\ 1 & -2 & 2 \\ 3 & -1 & 3 \end{pmatrix}^{-1} = \frac{1}{\begin{vmatrix} 2 & 1 & -1 \\ 1 & -2 & 2 \\ 3 & -1 & 3 \end{vmatrix}} \begin{pmatrix} \begin{vmatrix} -2 & 2 \\ -1 & 3 \end{vmatrix} & -1 \cdot \begin{vmatrix} 1 & -1 \\ -1 & 3 \end{vmatrix} & \begin{vmatrix} 1 & -1 \\ -2 & 2 \end{vmatrix} \\ -1 \cdot \begin{vmatrix} 1 & 2 \\ 3 & 3 \end{vmatrix} & \begin{vmatrix} 2 & -1 \\ 3 & 3 \end{vmatrix} & -1 \cdot \begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix} \\ \begin{vmatrix} 1 & -2 \\ 3 & -1 \end{vmatrix} & -1 \cdot \begin{vmatrix} 2 & 1 \\ 3 & -1 \end{vmatrix} & \begin{vmatrix} 2 & 1 \\ 1 & -2 \end{vmatrix} \end{pmatrix} \\ = \frac{1}{-10} \begin{pmatrix} -4 & -2 & 0 \\ 3 & 9 & -5 \\ 5 & 5 & -5 \end{pmatrix} = \begin{pmatrix} 2/5 & 1/5 & 0 \\ -3/10 & -9/10 & 1/2 \\ -1/2 & -1/2 & 1/2 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 & 1 & -1 \\ 1 & -2 & 2 \\ 3 & -1 & 3 \end{pmatrix}^{-1} \begin{pmatrix} 11 \\ 2 \\ 5 \end{pmatrix} = \begin{pmatrix} 2/5 & 1/5 & 0 \\ -3/10 & -9/10 & 1/2 \\ -1/2 & -1/2 & 1/2 \end{pmatrix} \begin{pmatrix} 11 \\ 2 \\ 5 \end{pmatrix} \\ = \begin{pmatrix} 11 \cdot 2/5 + 2 \cdot 1/5 + 5 \cdot 0 \\ 11 \cdot (-3/10) + 2 \cdot (-9/10) + 5 \cdot 1/2 \\ 11 \cdot (-1/2) + 2 \cdot (-1/2) + 5 \cdot 1/2 \end{pmatrix} = \begin{pmatrix} 24/5 \\ -13/5 \\ -4 \end{pmatrix}$$

Answer: $x=4.8$, $y=-2.6$, $z=-4$.