Question

Differentiate $y = (x^2 + sin(x))/(x + cos(x))$ with respect to x.

Solution

To calculate derivative of this function, we will use the quotient rule:

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2},$$

where $f'(x), g'(x)$ mean derivatives of $f(x)$ and $g(x)$ respectively.

Now we adapt this formula for our case:

$$\frac{dy}{dx} = \frac{(x + cos(x))(2x + cos(x)) - (x^2 + sin(x))(1 - sin(x))}{(x + cos(x))^2}$$

$$= \frac{2x^2 + 3xcos(x) + cos^2(x) - x^2 + x^2sin(x) - sin(x) + sin^2(x)}{(x + cos(x))^2}$$

$$= \frac{x^2 + 1 + 3xcos(x) + (x^2 - 1)sin(x)}{(x + cos(x))^2}.$$

Answer:

$$\frac{dy}{dx} = \frac{x^2 + 1 + 3xcos(x) + (x^2 - 1)sin(x)}{(x + cos(x))^2}.$$