Answer on Question #82784 – Math – Abstract Algebra

Question

Let R be an integral Domain then $deg(fg) = \deg(f) + \deg(g)$.

Solution

Let $\deg(f) = k$ and $\deg(g) = l$.

If $f = 0$ then $fg = 0$ too, and $k = -\infty$, so:

$$deg(fg) = -\infty = -\infty + l = \deg(f) + \deg(g)$$

So let's suppose that $f \neq 0$ and g$\neq 0$, so $k, l \geq 0$, so:

$$f = \sum_{i=0}^{k} f_i x^i \, , g = \sum_{j=0}^{l} g_j x^j$$

If $h = fg$, so $h_n = \sum_{i=0}^{n} f_i g_{n-i}$.

So, if $n > k + l$, then $h_n = 0$, because $f_i \neq 0$ only if $i \leq k$ and then $n - i > l$ so $g_{n-i} = 0$

But $h_{k+l} = f_k g_l \neq 0$ since $f_k \neq 0$, $g_l \neq 0$ and R is integral domain and it has no zero divisors.