Answer on Question \# 82783, Math / Differential Equations

Question 1. Let $u_{x x}+u_{y y}=Q(x, y)$ and $u(x, y)=c$. Assume $u=v+w$, where $v \rightarrow$ nonhomogeneous equation with homogeneous boundary conditions, $w \rightarrow$ homogeneous equation with non homogeneous boundary condition.

Solution. Let $\Delta u(x, y) \equiv u_{x x}(x, y)+u_{y y}(x, y)=Q(x, y),(x, y) \in \Omega$, where $\Omega$ is some region in the plane. If $Q(x, y) \not \equiv 0$ this equation is called the Poisson equation.

Let $u=v+w$ where $\Delta v(x, y)=f(x, y),(x, y) \in \Omega$ and $B v(x, y)=0,(x, y) \in \partial \Omega$; $\Delta w(x, y)=0,(x, y) \in \Omega$ and $B w(x, y)=g(x, y),(x, y) \in \partial \Omega$. General form for boundary condition:

$$
B u(x, y) \equiv \alpha(x, y) \frac{\partial u}{\partial n}(x, y)+\beta(x, y) u(x, y)=g(x, y),(x, y) \in \partial \Omega
$$

We have Dirichlet boundary condition: $B u(x, y)=u(x, y)=c$.

