Answer on Question # 82783, Math / Differential Equations

Question 1. Let $u_{xx} + u_{yy} = Q(x, y)$ and u(x, y) = c. Assume u = v + w, where $v \to non-homogeneous$ equation with homogeneous boundary conditions, $w \to homogeneous$ equation with non homogeneous boundary condition.

Solution. Let $\Delta u(x, y) \equiv u_{xx}(x, y) + u_{yy}(x, y) = Q(x, y), (x, y) \in \Omega$, where Ω is some region in the plane. If $Q(x, y) \neq 0$ this equation is called the Poisson equation.

Let u = v + w where $\Delta v(x, y) = f(x, y)$, $(x, y) \in \Omega$ and Bv(x, y) = 0, $(x, y) \in \partial \Omega$; $\Delta w(x, y) = 0$, $(x, y) \in \Omega$ and Bw(x, y) = g(x, y), $(x, y) \in \partial \Omega$. General form for boundary condition:

$$Bu(x,y) \equiv \alpha(x,y) \frac{\partial u}{\partial n}(x,y) + \beta(x,y)u(x,y) = g(x,y), \ (x,y) \in \partial \Omega.$$

We have Dirichlet boundary condition: Bu(x, y) = u(x, y) = c.