Answer on Question # 82704 - Math - Differential Equations

Question 1.

$$y^{(4)} + 2y^{(3)} - 3y'' = 3e^{2x} + 4\sin x$$

Answer.

$$y(x) = c_1e^{-3x} + c_2e^{x} + c_3x + c_4 + \frac{3}{20}e^{2x} + \frac{4}{5}\sin x + \frac{2}{5}\cos x,$$

where $c_1, c_2, c_3, c_4 \in \mathbb{R}$. □

Problem solving steps. 1) First, solve the corresponding homogeneous equation

$$y^{(4)} + 2y^{(3)} - 3y'' = 0:$$

characteristic equation $\lambda^4 + 2\lambda^3 - 3\lambda^2 = 0$, $\lambda_1 = -3, \lambda_2 = 1, \lambda_3 = \lambda_4 = 0$;
then solutions are $y_0(x) = c_1e^{-3x} + c_2e^{x} + c_3x + c_4$, where $c_1, c_2, c_3, c_4 \in \mathbb{R}$.
2) Next, set up a trial function (partial solution) $y_p(x) = ae^{2x} + b\sin x + c\cos x$ by copying the structure of RHS of the nonhomogeneous equation and substitute into the nonhomogeneous equation:

$$20ae^{2x} + (4b + 2c)\sin x + (4c - 2b)\cos x = 3e^{2x} + 4\sin x,$$

$$a = \frac{3}{20},\ b = \frac{4}{5},\ c = \frac{2}{5},$$

$$y_p(x) = \frac{3}{20}e^{2x} + \frac{4}{5}\sin x + \frac{2}{5}\cos x.$$

3) The general solutions to the nonhomogeneous equation are $y(x) = y_0(x) + y_p(x)$. □