

Question 1. Let $\phi(x, y, z) = xy^2z$ and $A(x, y, z) = xzi - xy^2j + xz^2$. Find $\frac{\partial^2}{\partial x \partial y}(\phi A)$ at the point $(2, -1, 1)$.

Solution.

$$\phi A = xy^2z(xzi - xy^2j + xz^2) = x^2y^2z^2i - x^2y^4zj + x^2y^2z^3$$

$$\partial_y(\phi A) = 2x^2yz^2i - 4x^2y^3zj + 2x^2yz^3$$

$$\partial_{xy}^2(\phi A) = \partial_x(2x^2yz^2i - 4x^2y^3zj + 2x^2yz^3) = 4xyz^2i - 8xy^3zj + 4xyz^3$$

$$\partial_{xy}^2(\phi A)\Big|_{(2,-1,1)} = 4 \cdot 2(-1)i - 8 \cdot 2(-1)^3j + 4 \cdot 2(-1) = -8i + 16j - 8$$

$$\frac{\partial^2}{\partial x \partial y}(\phi A)\Big|_{(2,-1,1)} = 8(-i + 2j - 1)$$

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