Answer on Question #82606 – Math – Abstract Algebra

Question

if a=b(mod r) and a=b(mod s) then a=b(mod [r,s])

Solution

We need to prove that if a=b(mod r) and a=b(mod s) then a=b(mod [r,s]) .

Proof

$$a = b(mod\ r) \Longleftrightarrow a = b + r \cdot k_1, \qquad \forall k_1 \in \mathbb{Z}$$

and $\exists l \in \mathbb{Z}: \ [r,s] = r \cdot l$.

We need to prove that

$$a = b(mod\ [r,s]) \Longleftrightarrow a = b + [r,s] \cdot k_2, \qquad \forall k_2 \in \mathbb{Z}$$

Just let $k_1 = l \cdot k_2$. The first formula is correct for all k_1, and then for $k_1 = l \cdot k_2$. Thus, the second formula is correct for all k_2.