

### Answer on Question #82599 – Math – Differential Equations

Solve the given DE or IVP. First you need to determine what type of DE it is.

$$y' = \sin^2(3x - 3y + 1)$$

### Solution

The first-order nonlinear ODE

$$\text{let } z(x) = 3x - 3y(x) + 1 \rightarrow \frac{dz(x)}{dx} = 3 - 3 \frac{dy(x)}{dx} \rightarrow \frac{dy(x)}{dx} = 1 - \frac{1}{3} \frac{dz(x)}{dx}$$

$$\text{then } 1 - \frac{1}{3} \frac{dz(x)}{dx} = (\sin(z(x)))^2$$

$$\frac{dz(x)}{(\cos(z(x)))^2} = 3 dx \rightarrow \int \frac{dz(x)}{(\cos(z(x)))^2} = \int 3 dx$$

$$\tan(z(x)) = \text{Const} + 3x$$

$$z(x) = \tan^{-1}(C + 3x) + k \cdot \pi, k = 0, \pm 1 \dots \pm \infty$$

$$\tan \alpha = \frac{2 \tan \alpha/2}{1 - (\tan \alpha/2)^2} \rightarrow \tan \alpha/2 = \frac{\sqrt{(\tan \alpha)^2 + 1} - 1}{\tan \alpha} \text{ and } \tan \alpha/2 = -\frac{\sqrt{(\tan \alpha)^2 + 1} + 1}{\tan \alpha}$$

$$\frac{\alpha}{2} = \tan^{-1} \left( \frac{\sqrt{(C + 3x)^2 + 1} - 1}{C + 3x} \right) \text{ or } \frac{\alpha}{2} = -\tan^{-1} \left( \frac{\sqrt{(C + 3x)^2 + 1} + 1}{C + 3x} \right)$$

$$z_1(x) = 2 \tan^{-1} \left( \frac{\sqrt{(C + 3x)^2 + 1} - 1}{C + 3x} \right), \quad z_2(x) = -2 \tan^{-1} \left( \frac{\sqrt{(C + 3x)^2 + 1} + 1}{C + 3x} \right)$$

$$\begin{aligned} z(x) = 3x - 3y(x) + 1 \rightarrow y_1(x) &= \frac{1}{3} + x + \frac{2}{3} \tan^{-1} \left( \frac{\sqrt{(C + 3x)^2 + 1} - 1}{C + 3x} \right), \quad y_2(x) \\ &= \frac{1}{3} + x - \frac{2}{3} \tan^{-1} \left( \frac{\sqrt{(C + 3x)^2 + 1} + 1}{C + 3x} \right) \end{aligned}$$

**Answer:**

$$y_1(x) = \frac{1}{3} + x + \frac{2}{3} \tan^{-1} \left( \frac{\sqrt{(C+3x)^2+1}-1}{C+3x} \right), \quad y_2(x) = \frac{1}{3} + x - \frac{2}{3} \tan^{-1} \left( \frac{\sqrt{(C+3x)^2+1}+1}{C+3x} \right)$$

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