

Answer to Question #82580 - Math / Abstract Algebra

**Question.** Prove that field has no zero divisors.

**Answer.** Let  $K$  be a field and  $a \in K$  be a zero divisor. By definition of zero divisor, there is  $b \in K \setminus \{0\}$  such that  $ab = 0$ , and  $a \neq 0$ . By definition of field, every non-zero element of  $K$  has an inverse, so there is  $b^{-1}$ . Multiplying  $ab = 0$  by  $b^{-1}$ , we have

$$a = abb^{-1} = 0 \cdot b^{-1} = 0,$$

contradiction. Therefore, there are no zero divisors in  $K$ .