Answer on Question #82566 – Math – Differential Equations

Question

Solve

$$\frac{d^2u}{dx^2}+\frac{d^2u}{dx^2}=0$$

$$y=0\,,y=10\,,x=infinity$$

$$u(x,y)=x-x^2\ at\ x\ =\ 10$$

Solution

$u=X(x)Y(y)$

$YX''+XY''=0$

$\frac{X''}{X}=-\frac{Y''}{Y}=\omega^2$

$\frac{d^2X}{dx^2}=\omega^2X\,;\,\frac{d^2Y}{dy^2}=-\omega^2Y$

The general solution is:

$u_k(x,y)=X_k(x)Y_k(y)=exp\left(-\frac{\omega_k x}{10}\right)\sin\frac{\omega_k y}{10},$

$u(x,y)=\sum_{k=1}^{\infty}a_kX_k(x)Y_k(y)=\sum_{k=1}^{\infty}a_k exp\left(-\frac{\omega_k x}{10}\right)\sin\frac{\omega_k y}{10}\ \ ;\ \ \omega_k=k\pi$

We have:

$u(10,y)=\sum_{k=1}^{\infty}a_ke^{-\omega_k}\sin\frac{\omega_k y}{10}=10-100=-90$

$$a_k=-90\cdot\frac{2}{10}\int_0^{10}e^{-\omega_k}\sin\frac{\omega_k y}{10}dy=18e^{-\omega_k}\cdot\frac{10}{\omega_k}(\cos\omega_k-1)=\frac{180e^{-\omega_k}}{\omega_k}((-1)^k-1)$$

Answer: $u(x,y) = 180\sum_{k=1}^{\infty}((-1)^k - 1)\frac{e^{-\omega_k}}{\omega_k} exp\left(-\frac{\omega_k x}{10}\right)\sin\frac{\omega_k y}{10}$.