ANSWER on Question #82560 – Math – Linear Algebra

QUESTION

Let

$$P^{(e)} = \{p(x) \in \boldsymbol{R}[x] | p(x) = p(-x)\}$$

Find $W = P^{(e)} \cap P_3$. Find a basis for W and find the dimension of W.

SOLUTION

P_3 consists of all polynomials of degree 3, i.e. of all polynomials like

$$p(x) = ax^3 + bx^2 + cx + d$$

Since $p(x) \in W \to \begin{cases} p(x) \in P_3 \\ p(x) \in P^{(e)} \end{cases}$. Then,

$$p(x) \in P^{(e)} \to p(x) = p(-x) \to$$

$$ax^3 + bx^2 + cx + d = a \cdot (-x)^3 + b \cdot (-x)^2 + c \cdot (-x) + d \to$$

$$ax^3 + bx^2 + cx + d = -ax^3 + bx^2 - cx + d \to$$

$$ax^3 + bx^2 + cx + d + ax^3 - bx^2 + cx - d = 0 \to$$

$$2ax^3 + 2cx = 0 \to \begin{cases} 2a = 0 \\ 2c = 0 \end{cases} \to \boxed{\begin{cases} a = 0 \\ c = 0 \end{cases}}$$

Conclusion,

$$\boxed{W = \{p(x) = b \cdot x^2 + d \cdot 1, \ \ \forall b, d \in \mathbb{R}\}}$$

$$\boxed{A\ basis\ for\ W\ is\ \{1, x^2\}}$$

Indeed, these two elements are linearly independent since

$$a \cdot 1 + b \cdot x^2 = 0 \ \ for\ all\ x \in \mathbb{R} \to \begin{cases} a = 0 \\ b = 0 \end{cases}$$

And all elements of W are linear combinations of 1 and x^2.

Hence, the dimension of W is $\boxed{\dim(W) = 2}$.

ANSWER:

$$W = \{p(x) = b \cdot x^2 + d \cdot 1,\ \ \forall b, d \in \mathbb{R}\}$$

$$A\ basis\ for\ W\ is\ \{1, x^2\}$$

$$\dim(W) = 2$$