Answer on Question #82545 – Math – Linear Algebra

Question

Show the cartesian product of R^n and R^m, is isomorphic to R^(n+m).

Solution

$R^n \times R^m = \{\langle (x_1, \dots, x_n), (y_1, \dots, y_m) \rangle | (x_1, \dots, x_n) \in R^n, (y_1, \dots, y_m) \in R^m\}$.

We will define function $f: R^n \times R^m \to R^{n+m}$ so that

$$f(\langle (x_1, \dots, x_n), (y_1, \dots, y_m) \rangle) = (x_1, \dots, x_n, y_1, \dots, y_m),$$

$(x_1, \dots, x_n) \in R^n$, $(y_1, \dots, y_m) \in R^m$, $\langle (x_1, \dots, x_n), (y_1, \dots, y_m) \rangle \in R^n \times R^m$ and $(x_1, \dots, x_n, y_1, \dots, y_m) \in R^{n+m}$.

Then *f* is isomorphism.

Let $(x_1, \dots, x_n, x_{n+1}, \dots, x_{n+m}) \in R^{n+m}$ then$(x_1, \dots, x_n) \in R^n$,$(x_{n+1}, \dots, x_{n+m}) \in R^m$, $\langle (x_1, \dots, x_n), (x_{n+1}, \dots, x_{n+m}) \rangle \in R^n \times R^m$ and $f(\langle (x_1, \dots, x_n), (x_{n+1}, \dots, x_{n+m}) \rangle) = (x_1, \dots, x_n, x_{n+1}, \dots, x_{n+m})$ therefore *f* is function onto.

Let $\langle (x_1, \dots, x_n), (x_{n+1}, \dots, x_{n+m}) \rangle, \langle (x'_1, \dots, x'_n), (x'_{n+1}, \dots, x'_{n+m}) \rangle \in R^n \times R^m$ and $\langle (x_1, \dots, x_n), (x_{n+1}, \dots, x_{n+m}) \rangle \neq \langle (x'_1, \dots, x'_n), (x'_{n+1}, \dots, x'_{n+m}) \rangle$ then $\exists i (1 \leq i \leq n + m$ & $x_i \neq x'_i)$ but then $(x_1, \dots, x_n, x_{n+1}, \dots, x_{n+m}) \neq (x'_1, \dots, x_n, x_{n+1}, \dots, x_{n+m})$ that is $f(\langle (x_1, \dots, x_n), (x_{n+1}, \dots, x_{n+m}) \rangle) \neq f(\langle (x'_1, \dots, x'_n), (x'_{n+1}, \dots, x'_{n+m}) \rangle)$.

So, *f* is one-to-one function.