Answer on Question #82544 – Math – Linear Algebra

Question

Let $V = \boldsymbol{R}^3$, $A = \{(x, y, z)|y = 0\}$ and $B = \{(x, y, z)|x = y = z\}$.

Check whether $\boldsymbol{R}^3 = A \oplus B$.

Solution

$C = A \oplus B$ if and only if $C = A + B$ and $A \cap B = 0$.

1) $(x, y, z) \in A \cap B$ if and only if $(x, y, z) \in A$ and $(x, y, z) \in B$, so

If $(x, y, z) \in B$ then $x = y = z$ so $(x, y, z) = (x, x, x)$ and if $(x, x, x) \in A$ then $x = 0$ so $(x, y, z) = (0,0,0)$, so $A \cap B = 0$

2) $(x, y, z) = (x - y + y, y, z - y + y) = (x - y, 0, z - y) + (y, y, y)$

$(x - y, 0, z - y) \in A$ and $(y, y, y) \in B$ so $\boldsymbol{R}^3 = A + B$

So, both conditions are true and $\boldsymbol{R}^3 = A \oplus B$.

Answer: $\boldsymbol{R}^3 = A \oplus B$ holds true.