Answer on Question #82539 – Math – Abstract Algebra

Question

Show that d:QQ[x]\{0}→NN∪ {0}:d(f)=2^(deg f) is a Euclidean valuation on QQ[x].

Solution

By the definition, we need to show that

1) d(f)<=d(fg);
2) $\forall\, f, g \in Q[x]\ \exists\, q, r \in R: f = qg + r$ and either r=0 or d(r)<d(g).

1. deg(f) < deg(fg)=deg(f)+deg(g) , hence $d(f) = 2^{deg(f)} <= 2^{deg(fg)} = d(fg)$
2. There is division with remainder in Q[x]
   $\forall\, f, g \in Q[x]\ \exists\, q, r \in R: f = qg + r$ such that either r=0 or deg(r)<deg(g).
   Thus, $\forall\, f, g \in Q[x]\ \exists\, q, r \in R: f = qg + r$ such that either r=0 or
   $d(r) = 2^{deg(r)} < 2^{deg(g)} = d(g)$.